AQ–2927

Faculty of Engineering & Technology

M.E. Semester—II Examination

ADAPTIVE SIGNAL PROCESSING

Paper—2 ENTC 1

Sections—A & B

Time : Three Hours] [Maximum Marks : 80

INSTRUCTIONS TO CANDIDATES

(1) All questions carry marks as indicated.

(2) Answer THREE questions from Section A and THREE questions from Section B.

(3) Due credit will be given to neatness and adequate dimensions.

(4) Assume suitable data wherever necessary.

(5) Use pen of Blue/Black ink/refill only for writing the answer book.

SECTION—A

1. (a) The sequences y(n) and u(n) are related by the difference equation

$$y(n) = u(n + a) - u(n - a)$$

where a is a constant. Evaluate the autocorrelation function of y(n) in terms of that of u(n). 8

(b) State and explain any three properties of the Correlation Matrix. 6

OR

2. (a) Consider an autoregressive process u(n) of order two described by the difference equation, $u(n) = u(n-1) - 0.5\,u(n-2) + v(n)$ where v(n) is white noise with zero mean and variance 0.5.

(i) Write the Yule-Walker equations for the process.

(ii) Solve these two equations for autocorrelation function values r(1) and r(2).

(iii) Find the variance of u(n). 8

(Contd.)

(b) Consider a correlation matrix R for which the inverse matrix R^{-1} exists. Show that R^{-1} is Hermitian. 6

3. (a) Consider a Wiener filtering problem characterized as follows. The correlation matrix R of the tap-input vector u(n) is :

$$R = \begin{bmatrix} 1 & 0.5 \\ 0.5 & 1 \end{bmatrix}.$$

The cross-correlation vector between the tap-input vector u(n) and the desired response d(n) is $p = [0.5, 0.25]^T$

(i) Evaluate the tap weights of the Wiener filter.

(ii) What is the minimum mean-square error produced by this Wiener filter ?

(iii) Formulate a representation of the Wiener filter in terms of the eigen values of matrix R and associated eigen vectors. 8

(b) Explain solution of the Wiener-Hopf equations for linear transversal filter. 5

OR

4. (a) Suppose you are given the two time series u(0), u(1), ..., u(N) and d(0), d(1), ..., d(N) both of which are realizations of two jointly wide-sense stationary processes. The series are used to supply the tap inputs of a transversal filter of length M and the desired response, resp. Assuming that both of these processes are jointly ergodic, derive an estimate for the tap-weight vector of the Wiener filter by using time averages. 6

(b) The statistical characterization of a multiple linear regression model of order four is as follows :

The correlation matrix of the input vector u(n) is :

$$R_4 = \begin{bmatrix} 1.1 & 0.5 & 0.1 & -0.1 \\ 0.5 & 1.1 & 0.5 & 0.1 \\ 0.1 & 0.5 & 1.1 & 0.5 \\ -0.1 & 0.1 & 0.5 & 1.1 \end{bmatrix}$$

The cross correlation vector between the observable data and the input vector is

$$p_4 = [0.5, -0.4, -0.2, -0.1]^T.$$

The variance of the observable data d(n) is :

$\sigma_d^2 = 1.0.$

The variance of the additive white noise is :

$\sigma_d^2 = 0.1.$

A Wiener filter of varying length M operates on the input vector u(n) as input and on the observable data d(n) as the desired response. Compute and plot the mean-square error produced by the Wiener filter of M = 0, 1, 2, 3, 4. 7

5. (a) The steepest descent algorithm becomes unstable when the step-size parameter μ is assigned a negative value. Justify the validity of this statement. 6

(b) Consider the use of a white-noise sequence of zero mean and variance σ^2 as the input to the LMS algorithm. Evaluate :

(i) The condition for convergence of the algorithm in the mean square.

(ii) The excess mean-square error. 7

OR

6. (a) In the method of steepest descent, show that the correction applied to tap-weight vector after n + 1 iterations may be expressed as :

$$\delta w(n+1) = \mu E[u(n)e^*(n)],$$

where $\mu(n)$ is the tap-input vector and e(n) is the estimation error. What happens to this adjustment at the minimum point of the error-performance surface ? 7

(b) The LMS algorithm is used to implement a dual input, single-weight adaptive noise canceller. Set up the equations that define the operation of this algorithm. 6

SECTION—B

7. (a) State and explain the three important properties of the innovation $\alpha(n)$. 6

(b) Establish the validity of the matrix inversion lemma. 7

OR

8. Explain in detail Kalman filtering. 13

9. (a) Give some examples of applications where adaptive filtering is done. 6

(b) Explain convergence analysis of the RLS algorithm. 7

OR

10. Derive the mean square error in RLS algorithm with optimization. 13

11. (a) What is the need for adaptive equalization in a digital communication system ? Explain. 7

(b) Discuss the gradient adaptive lattice algorithm. 7

OR

12. (a) How is the effect of echo minimized in a telephone communication ? 7

(b) Explain with a neat labelled diagram the removal of ocular artifacts from ECG using adaptive filtering. 7