. First Semester M. E. (Electrical and Elect.) (New CGS) Examination

## ADVANCED DIGITAL SIGNAL PROCESSING

Paper - 1 EEEME 3
P. Pages: 6

Time : Three Hours $]$
[ Max. Marks : 80
Note : (1) All question carry marks as indicated.
(2) Assume suitable data wherever necessary.
(3) Illustrate your answer wherever necessary with the help of neat sketches.
(4) Use pen of Blue/Black ink/refill only for writing the answer book.

1. (a) A linear time-invariant system with frequency response $H(w)$ excited with the periodic input.

$$
\mathrm{x}(\mathrm{n})=\sum_{\mathrm{k}=-\infty}^{\infty} \delta(\mathrm{n}-\mathrm{KN})
$$

Suppose that we conpute the $N$-point DFT $Y(K)$ of the samples $y(n)$, $0 \leqslant n \leqslant N-1$ of the output sequence. How is $Y(K)$ related to $H(W)$ ?
(b) Compute the eight point. DFT of the sequence $x(n)=\{1 / 2,1 / 2,1 / 2,1 / 2,0,0,0,0\}$
Using the in-place radix-2 decimation-in-frequency algorithm. Follow exactly the corresponding signal flow graph and keep track of all the intermediate quantities by putting them on the diagram.

## OR

2. (a) Determine the response $y(n),-n \geqslant 0$, of the system described by the second-order difference equation $y(n)-3 y(n-1)-4 y(n-2)=x(n)+$ $2 x(n-1)$ to the input $x(n)=4^{n} u(n)$.
(b) A discrete-time system is realized by the structure shown in figure Q. 2 b .
[Fig on Next page]

(i) Determine the impulse response.
(ii) Determine a realization for its inverse system, that is, the system which produces $x(n)$ as an output when $y(n)$ is used as an input.
3. (a) The first-order filter shown in figure Q . Ba is implemented in four-bit (including sign) fixed-point Two's-complement fractional arithmetic. Products are rounded to four-bit respresentation. Using the input $x(n)=0.10 \delta(n)$,


Determine :-
(i) The first five outputs if $\alpha=0.5$. Does the filter go into a limit cycle?
(ii) The first five outputs if $\alpha=0.75$. Does the filter go into a limit cycle?
(b) Obtain the direct form I, direct form II, cascade and parallel structures for the following systems :

$$
\begin{align*}
& y(n)=-0.1 y(n-1)+0.2 y(n-2)+3 x(n) \\
& +3.6 x(n-1)+0.6 x(n-2) \tag{7}
\end{align*}
$$

## OR

4. (a) Consider the system :

$$
y(n)=0.875 y(n-1)-0.125 y(n-2)+x(n)
$$

(i) Compute its poles and design the cascade realization of the system.
(ii) Quantize the coefficients of the system using function, maintaining a sign bit plus three other bits. Determine the poles of the resulting system.
(iii) Repeat part (ii) for the same precision using rounding.
(b) Determine a parallel and a cascade realization of the system :

$$
\begin{equation*}
H(z)=\frac{1+z^{-1}}{\left(1-z^{-1}\right)\left(1-0.8 e^{j / 4} z^{-1}\right)\left(1-0.8 e^{-5 / 4 / 4} z^{-1}\right)} \tag{6}
\end{equation*}
$$

5. (a) Determine the system function $\mathrm{H}(\mathrm{z})$ of the lowest-order Chebyshev digital filter that meets the following specifications :
(i) $\frac{1}{2} \mathrm{~dB}$ ripple in the
passband $0 \leqslant|w| \leqslant 0.24 \pi$
(ii) At least 50 dB attenuation in the stopband $0.35 \pi \leqslant|\omega| \leqslant \pi$. Use the bilinear transformation.
(b) Determine the coefficients $\{\mathrm{h}(\mathrm{n})$ \} of a linear-phase FIR filter of length $\mathrm{m}=15$ which has a symmetric unit sample response and a frequency response that satisfies the condition :

$$
\operatorname{Hr}\left(\frac{2 \pi \mathrm{k}}{15}\right)= \begin{cases}1, & k=0,1,2,  \tag{6}\\ 0, & k=4,5,6,7\end{cases}
$$

## OR

6. (a) An analog signal of the form $x_{2}(t) \quad=a(t) \cos 2000 \pi t$ is bandlimited to the range $900 \leqslant F \leqslant 1100 \mathrm{~Hz}$. It is used as an input to the system shown in figure Q. 6a.

(i) Determine and sketch the spectra for the signals $x(n)$ and $w(n)$.
(ii) Use a Hamming window of length $m=31$ to design a lowpass linear phase FIR filter $H(w)$ that passes $\{a(n)\}$.
(iii) Determine the sampling rate of theA/D converter that would allow us to eliminate the frequency conversion in figure Q.6a.
(b) Determine the system function $\mathrm{H}(\mathrm{z})$ of the lowest order Chebyshev digital filter that meets the following specifications :
(i) 1 dB ripple in the passband $0 \leqslant|\omega| \leqslant 0.3 \pi$.
(ii) At least 60 dB attenuation in the stop band $0.35 \pi \leqslant|w| \leqslant \pi$. Use bilinear trans-formation.
7. (a) An $\mathrm{mA}(2)$ process has the autocorrelation sequence :

$$
v_{x x}(m)=\left\{\begin{array}{cl}
6 \sigma_{w}^{2}, & m=0 \\
-4 \sigma_{w}^{2}, & m= \pm 1 \\
-2 \sigma_{\omega}^{2}, & m= \pm 2 \\
0, & \text { otherwise }
\end{array}\right.
$$

(i) Determine the coefficients of the $\mathrm{mA}(2)$ process that have the foregoing autocorrelation.
(ii) Is the solution unique ? If not, give all the possible solutions. 7
(b) Show that the periodogram values at frequencies $f_{k}=k / L, k=0,1, \cdots$. L-I, given by

$$
P_{x x}\left(\frac{k}{L}\right)=\frac{1}{L}\left|\sum_{n=0}^{L=1} x(n) e^{-j 2 \pi k / L}\right|^{2}, k=0,1,--, L=1
$$

Can be computed by passing the sequence through a bank of N IIR filter, where each filter has an impulse response :

$$
h k(n)=e^{-j 2 n_{k} K / N} u(n)
$$

and then compute the magnitude-squared value of the filter outputs at $n=N$. Each filter has a pole on the unit circle at the frequency $f_{k}$. 6

## OR

8. (a) Suppose we have $\mathrm{N}=1000$ samples from a sample sequence of a random process.
(i) Determine the frequency resolution of the Bartlett, Welch ( $50 \%$ overlap), and Blackman-Tukey methods for a quality factor $\mathrm{Q}=10$.
(ii) Determine the record lengths (M) for the Bartlett, Welch ( $50 \%$ overlap) and Blackman-Tukey methods.
(b) Determine the power spectra for the random process generated by the following difference equations :
(i) $\mathrm{x}(\mathrm{n})=-0.81 \mathrm{x}(\mathrm{n}-2)+\mathrm{w}(\mathrm{n})-\mathrm{w}(\mathrm{n}-1)$
(ii) $x(n)=w(n)-w(n-2)$
(iii) $x(n)=-0.81 x(n-2)+w(n)$

Where $\mathrm{w}(\mathrm{n})$ is a white noise process with variance
Sketch the spectra for the processes given in (i), (ii), and (iii).
9. (a) A sequence $\mathrm{x}(\mathrm{n})$ is upsampled by $\mathrm{I}=2$, it passes through an LTI system $\mathrm{H}_{\mathrm{i}}(\mathrm{z})$, and then it is downsampled by $\mathrm{D}=2$. Can we replace this process with a single LTI system $\mathrm{H}_{\mathbf{2}}(\mathrm{z})$ ? If the answer is positive, determine the system function of this system.
(b) Show that the transpose of an L -stage interpolator for increasing the sampling rate by an integer factor $I$ is equivalent to an $L$-state decimator that decreases the sampling rate by a factor $D=I$.
10. (a) Design an interpolator that increases the input sampling rate by a factor of $I=2$. Use the Remez algorithm to determine the coefficients of the FIR filter that has a 0.1 dB ripple in the passband $(0 \leqslant \mathrm{w} \leqslant \pi / 2)$ and is down by at least 30 dB in the stopband. Also, determine the corresponding polyphase filter structure for implementing the interpolator. 7 .
(b) We wish to design an efficient non recursive decimator for $D=8$ using factorization :
$H(z)=\left[\left(1+z^{-1}\right)\left(1+z^{-2}\right)\left(1+z^{-4}\right)-\left(1+z^{-2^{k-1}}\right)\right]^{5}$
(i) Derive an efficient implementation using filters with system function $H_{k}(z)=\left(1+z^{-1}\right)^{5}$
(ii) Show that each stage of the obtained decimator can be implemented more efficienntly using a polyphase decom-position.
11. (a) Discuss the implementation of DSP algorithms on general purpose processor.
(b) Discuss special purpose DSP processes for digital filters. 7 OR
12. (a) Discuss advantages and disadvantages of general purpose DSP. 7
(b) Discuss special purpose DSY processes for FFT. 7

