AQ – 2749

First Semester M. Tech. (Chemical Engineering) Examination

MATHEMATICAL MODELLING AND OPTIMIZATION

ICER

P. Pages : 2

Time : Three Hours] [Max. Marks : 80

Note : (1) All questions carry marks as indicated.
(2) Answer any **Six** questions.
(3) Assume suitable data wherever necessary.
(4) Illustrate your answer wherever necessary with the help of neat sketches.
(5) Use pen of Blue/Black ink/refill only for writing the answer book.

1. Find the dimensions of a cylindrical vessel with top and bottom, made-up of sheet metal to maximize its volume such that the total surface area is equal to 24π. Solve the problem by Lagrange multiplier method and also apply necessary and sufficient condition to the function.

2. A pharmaceutical company has 100 kg of A, 180 kg of B and 120 kg of C ingredients available per month. The company makes three products using these ingrediants, namely 5-10-5, 5-5-10 and 20-5-10. The numbers indicate percentage weight of A, B, and C. Respectively. The cost of these raw materials A, B, C and inerts are Rs. 80, 20, 50 and 20 respectively, while the selling price of the products are Rs. 40.50, Rs. 43 and Rs. 45 per kg respectively. There is a restriction on the production capacity of product 5-10-5, and the company cennot produce more than 30 kg per month. Determine how much of these products the company should produce to maximize the profits monthly.

3. Minimize $Z = 600x_1 + 500x_2$
Subject to $2x_1 + x_2 \geq 80$; $x_1 + 2x_2 \geq 60$
and $x_1\ x_2 \geq 0$. Use penalty method to solve the problem.

4. Give a critical account of the following :—

   (a) Concepts of reproduction, cross over and mutation in genetic algorithms.

   (b) Representation of objective function with constraints and design variables.

   (c) Typical architecture of a neural network.

5. Discuss in depth the assumptions involved, requirement, limitations and advantages of linear programming problems.

6. Find all the basic solutions of the following problem :—

Maximize $Z = x_1 + 3x_2 + 3x_3$
Subject to $x_1 + 2x_2 + 3x_3 = 4$
$2x_1 + 3x_2 + 5x_3 = 7$

Also find which of the basic solutions are

(i) Basic feasible.

(ii) Non-degenerate basic feasible and

(iii) Optimal basic feasible.

7. Show that the following f(x) is convex by expressing it as a sum of functions of one or two variables, and then prove that all the functions are convex. f $(x)=5x_1+2x_2^2+x_3^2-3x_3x_4+4x_4^2+2x_5^4+x_5^2+3x_5x_6+6x_6^2+3x_6x_7+x_7^2$

8. Give a description of varioous search methods like grid search, simplex search, random search with their important features used in optimization problem.

9. Show that a linear programming problem may have (i) No solution (ii) Unbounded solution, (iii) a single optimum solution or (iv) an infinite number of optimal solutions, by considering the geometry of the following example.

Maximize $f = x_1 + 3x_2$
Subject to $-x_1 + x_2 \leq 1$
$x_1 + x_2 \leq 2$
$x_1 \geq 0,\ x_2 \geq 0$

10. Give an informative account of the following :

(a) Types and classification of integer programming problems.

(b) Concepts of stage, state and return function and their functional relationship in dynamic programming.