M.E. First Semester (Mechanical Engineering (Thermal Engineering)) (New-CGS)

13505 : Advanced Mathematics : 1 MTE 1

P. Pages : 2
Time : Three Hours

AW - 3528
Max. Marks : 80

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Notes : 1. All question carry equal marks.
2. Answer **two** question from Section A and **two** question from Section B.
3. Due credit will be given to neatness and adequate dimensions.
4. Assume suitable data wherever necessary.
5. Use of slide rule logarithmic tables, normal table, calculator is permitted.
6. Use of pen Blue/Black ink/refill only for writing the answer book.

SECTION – A

1. a) Solve $\frac{\partial^2 z}{\partial x^2}+3\frac{\partial^2 z}{\partial x\,\partial y}+2\frac{\partial^2 z}{\partial y^2}=x+y$ **6**

b) $(D^3-3DD'^2-2D'^3)\,Z=\cos(x+2y)-e^y(3+2x)$ **8**

c) $(D^3-7DD'^2-6D'^3)\,Z=\sin(x+2y)+x^2y$ **6**

2. a) Solve the equation by the method of separation of variables. **7**
$\frac{\partial u}{\partial x}=4\frac{\partial u}{\partial y}$, given that $u(0,y)=8e^{-3y}$

b) A tightly stretched flexible string has its ends fixed at $x=0$ and $x=\ell$. At time $t=0$, the string is given a shape defined by $F(x)=\mu x(\ell-x)$, where μ is a constant and then released. Find the displacement of any point x of the string at any time $t>0$. **7**

c) Solve $\frac{\partial^3 z}{\partial x^3}-2\frac{\partial^3 z}{\partial x^2\partial y}=2e^{2x}+3x^2y$ **6**

3. a) Solve the differential equation $\frac{\partial u}{\partial t}=\alpha^2\frac{\partial^2 u}{\partial x^2}$ for the conduction of heat along a rod without radiation, subject to the following conditions. **7**
i) u is not infinite for $t\to\infty$.
ii) $\frac{\partial u}{\partial x}=0$ for $x=0$ and $x=\ell$.
iii) $u=\ell x-x^2$ for $t=0$, between $x=0$ and $x=\ell$.

b) A rectangular plate with insulated surfaces is 10 cm wide and so long compared to its width that it may be considered infinite in length without introducing an appreciable error. If the temperature along the short edges are kept at 0°C. Find the steady state temperature at any point (x, y) of the plate. **7**

c) Using the method of separation of variable, solve **6**
$\frac{\partial u}{\partial t}=\frac{\partial^2 u}{\partial x^2}$ if $u(x,0)=2x$ when $0\le x\le\frac{\ell}{2}=2(\ell-x)$ when $\frac{\ell}{2}\le x\le\ell$

SECTION – B

4. a) The following values of x and y are supposed to follow the law $y = ax^2 + b \log_{10} x$. Find graphically the most probable values of the constants a and b. 7

x	2.85	3.88	4.66	5.69	6.65	7.77	8.67
y	16.7	26.4	35.1	47.5	60.6	77.5	93.4

b) A restaurant serves two special dishes, A and B to its customers consisting of 60% men and 40% women. 80% of men order dish A and the rest B. 70% of women order B and the rest A. In what ratio of A to B should the restaurant prepare the two dishes? 6

c) A sales tax officer has reported that the average sales of the 500 businesses that he has to deal with during a year amount to Rs. 36,000 with a standard deviation of Rs. 10,000. Assuming that the sales in these businesses are normally distributed, find 7

i) The number of businesses, the sales of which are over Rs. 40,000/-

ii) The percentage of businesses, the sales of which are likely to range between Rs. 30,000 and Rs. 40,000.

iii) The probability that the sales of a businesses selected at random will be over Rs. 30,000 proportion of Area under the normal curve:

Z	0.25	0.40	0.50	0.60
Area	0.0987	0.1554	0.1915	0.2257

5. a) In the table below, the values of y are consecutive terms of a series of which 23.6 is the 6^{th} term. Find the first and tenth terms of the series using Newtons interpolation formula. 7

x	3	4	5	6	7	8	9
y	4.8	8.4	14.5	23.6	36.2	52.8	73.9

b) Using Gauss's backward formula estimate the number of persons earning wages between Rs. 60 and Rs. 70 from the following data. 6

Wages (Rs.)	Below 40	40 – 60	60 – 80	80 – 100	100 – 120
No. of persons (In thousands)	250	120	100	70	50

c) Using Lagrange's interpolation, calculate the profit in the year 2000 from the following data:

Year	1997	1999	2001	2002
Profit in Lakhs of Rs.	43	65	159	248

6. a) Given that 10

x	4.0	4.2	4.4	4.6	4.8	5.0	5.2
log x	1.3863	1.4351	1.4816	1.5261	1.5686	1.6094	1.6487

evaluate $\int_4^{5.2} \log x \, dx$ by

i) Trapezoidal rule ii) Simpson's $\frac{3}{8}^{th}$ rule

b) Given $\frac{dy}{dx} = x(x^2 + y^2)e^{-x}, y(0) = 1,$ 10

find y at x = 0.1, 0.2 and 0.3 by Taylor's series method and compute y(0.4) by Milne's method.

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