B.Sc. Part-III (Semester-VI) Examination

MATHEMATICS

(Special Theory of Relativity)

Paper—XII

Time : Three Hours] [Maximum Marks : 60

Note :—(1) Question No. **1** is compulsory, attempt once.

(2) Attempt **ONE** question from each unit.

1. Choose the correct alternative :

(i) The interval $ds^2 = -(dx^1)^2 - (dx^2)^2 - (dx^3)^2 + (dx^4)^2$ is said to be space like if : 1

(a) $ds^2 > 0$ (b) $ds^2 < 0$

(c) $ds^2 = 0$ (d) None of these

(ii) The electric and magnetic field strengths E and H are invariant under : 1

(a) Galilean Transformations (b) Laplace Transformations

(c) Fourier Transformations (d) Guage Transformations

(iii) $A^i = (\overline{A}, \phi) = (Ax, Ay, Az, \phi)$ is a four potential then : 1

(a) $A_i = (\overline{A}, \phi)$ (b) $A_i = (\overline{A}, -\phi)$

(c) $A_i = (-\overline{A}, \phi)$ (d) $A_i = (-\overline{A}, -\phi)$

(iv) $A^r = (A^1, A^2, A^3, A^4)$ is a four vector or four dimensional vector where $A^2 < 0$ then A^r is : 1

(a) Time like (b) Null or light like

(c) Space like (d) None of these

(v) Covariant tensor of rank one T'_r is defined as : 1

(a) $T'_r = \dfrac{\partial x'^r}{\partial x^5} T_s$ (b) $T'_r = \dfrac{\partial x'^r}{\partial x^5} T_r$

(c) $T'_r = \dfrac{\partial x^5}{\partial x'^r} T_s$ (d) $T'_r = \dfrac{\partial x^5}{\partial x'^r} T_r$

(vi) The special Lorentz transformations will reduce to simple Galilean transformations when : 1

(a) V = C (b) C < < V

(c) V < < C (d) None of these

(vii) The electromagnetic field tensor (or Maxwell tensor) F_{ij} is defined as : 1

(a) $F_{ij} = \frac{\partial A_i}{\partial x^j} - \frac{\partial A_j}{\partial x^i}$ (b) $F_{ij} = \frac{\partial A_j}{\partial x^i} - \frac{\partial A_i}{\partial x^j}$

(c) $F_{ij} = \frac{\partial A_i}{\partial x^j} + \frac{\partial A_j}{\partial x^i}$ (d) None of these

(viii) The transformations $\bar{r}' = \bar{r} - \bar{v}t$ and $t' = t$ are : 1

(a) Laplace transformations (b) Lorentz transformations

(c) Galilean transformations (d) None of these

(ix) If $\bar{A}$ is a vector potential then the magnetic field is : 1

(a) $\bar{H} = \text{div}.\bar{A}$ (b) $\bar{H} = \text{Curl}\,\bar{A}$

(c) $\bar{H} = \text{div}.(\text{Curl}\,\bar{A})$ (d) None of these

(x) Four velocity of a particle is : 1

(a) a unit space-like vector (b) a unit time-like vector

(c) a unit light-like vector (d) None of these

UNIT—I

2. (a) Obtain Galilean transformation equations for two inertial frames in relative motion. 3

(b) Show that simultaneity is relative in special relativity. 3

(c) Show that the electromagnetic wave equation :

$$\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} + \frac{\partial^2 f}{\partial z^2} - \frac{1}{c^2}\frac{\partial^2 f}{\partial t^2} = 0$$

is not invariant under the Galilean transformations. 4

3. (p) Discuss the geometrical interpretation of Lorentz transformations. 4

(q) Prove that $\nabla^2 - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}$ is invariant under special Lorentz transformations. 4

(r) Show that $x^2 + y^2 + z^2 - c^2t^2$ is Lorentz invariant. 2

UNIT—II

4. (a) Obtain the transformations for the velocity of a particle under special Lorentz transformations. 5

(b) If $\bar{u}$ and $\bar{u}'$ be the velocities of a particle in two inertial systems s and s' respectively where s' is moving with velocity v relative to s along the XX' axis then show that :

$$\tan\theta' = \frac{\sin\theta\left(1-\frac{v^2}{c^2}\right)^{1/2}}{\cos\theta - \frac{v}{u}}$$

and

$$u'^2 = \frac{u^2\left[1-2\frac{v}{u}\cos\theta+\frac{v^2}{u^2}-\frac{v^2}{c^2}\sin^2\theta\right]}{\left(1-\frac{uv}{c^2}\cos\theta\right)^2}$$

where θ and θ' are the angles made by u and u' with the X-axis respectively. 5

5. (p) If $\bar{u}$ and $\bar{u}'$ be the velocities of a particle in two inertial systems s and s' respectively then prove that :

$$\sqrt{1-\frac{u^2}{c^2}} = \frac{\sqrt{1-\frac{u'^2}{c^2}}\sqrt{1-\frac{v^2}{c^2}}}{\left(\frac{1+u'_x v}{c}\right)},$$

where s' is moving with velocity v relative to s along XX' axis. 5

(q) Show that in nature no signal can move with a velocity greater than the velocity of light relative to any inertial system. 5

UNIT—III

6. (a) Define time-like, space-like and light-like intervals for the space time geometry of special relativity. 3

(b) Define a four tensor of the second order. Prove that :

(i) $T'^{11} = \alpha^2\left\{T^{11} - \frac{v}{c}T^{14} - \frac{v}{c}T^{41} + \frac{v^2}{c^2}T^{44}\right\}$ and

(ii) $T'^{14} = \alpha^2\left\{-\frac{v}{c}T^{11} + T^{14} + \frac{v^2}{c^2}T^{41} - \frac{v}{c}T^{44}\right\}$ 1+3+3

7. (p) Define a four vector A^r. Show that :

$A^1 = -A_1$, $A^2 = -A_2$, $A^3 = -A_3$, $A^4 = A_4$. 1+3

(q) Prove that there exists an inertial system s' in which the two events occur at one and the same time if the interval between two events is space-like. 4

(r) Write the Lorentz transformations in index form. 2

UNIT—IV

8. (a) Deduce Einstein's mass energy equivalence relation. 5

(b) Define : Four velocity. Prove that the four velocity in component form can be expressed as :

$$u^i = \left(\frac{\bar{u}}{c\sqrt{1-u^2/c^2}}, \frac{1}{\sqrt{1-u^2/c^2}} \right)$$

where $\bar{u} = (u_x, u_y, u_z)$ – velocity of the particle. 1+4

9. (p) Define : Four momentum vector p^i. Prove that the square of the magnitude of the four momentum vector p^i is $m^2{}_\circ c^2$. 1+4

(q) A particle is given a kinetic energy equal to n times its rest energy $m_\circ c^2$. Find speed and momentum of the particle. $\left(\text{Kinetic energy} = T = m_\circ c^2 \left\{ \frac{1}{\sqrt{1-v^2/c^2}} - 1 \right\}\right)$ 5

UNIT—V

10. (a) Show that the Hamiltonian for a charged particle moving in an electromagnetic field is :

$$H = \left\{ m^2c^4 + c^2\left(p - \frac{e}{c}A \right)^2 \right\}^{1/2} + e\phi .$$ 5

(b) Define : Current four vector. Show that $c^2\rho^2 - J^2$ is invariant and its value is $\rho^2{}_\circ c^2$. 1+4

11. (p) Prove that the set of Maxwell's equations $\operatorname{div}.\bar{H} = 0$ and $\bar{E} = -\frac{1}{c}\frac{\partial \bar{H}}{\partial t}$ can be written as $\frac{\partial F_{ij}}{\partial x^k} + \frac{\partial F_{jk}}{\partial x^i} + \frac{\partial F_{ki}}{\partial x^j} = 0$, where F_{ij} is the electro-magnetic field tensor. 5

(q) Define electromagnetic field tensor F_{ij}. Express the components of F_{ij} in terms of the electric and magnetic field strengths. 1+4