AU–159

B.Sc. (Part-III) Semester-VI Examination
MATHEMATICS (New)
(Special Theory of Relativity)
Paper—XII

Time : Three Hours] [Maximum Marks : 60

Note :— (1) Question No. **1** is compulsory.

(2) Attempt **ONE** question from each unit.

1. Choose the correct alternative :

(i) If ds = 0, then the interval ds is said to be : 1

(a) light like (b) space like

(c) time like (d) None of these

(ii) Lorentz transformation reduces to Galilean transformation if : 1

(a) V = C (b) V > > C

(c) V < < C (d) None of these

(iii) Signature of the Minkowskian space-time $ds^2 = -dx^2 - dy^2 - dz^2 + c^2dt^2$ is : 1

(a) 2 (b) –2

(c) 3 (d) 1

(iv) The transformations $\bar{r}' = \bar{r} - \bar{v}t$ and t' = t are known as : 1

(a) General Lorentz transformation (b) Special Lorentz transformation

(c) Simple Galilean transformation (d) General Galilean transformation

(v) The time recorded by a clock moving with a body is known as : 1

(a) Time dilation (b) Proper time

(c) Fixed line (d) None of these

(vi) The simultancity in special relativity is : 1

(a) relative (b) constant

(c) absolute (d) None of these

(vii) The four velocity of a particle is a unit ______ vector. 1

(a) space like
(b) light like
(c) time like
(d) None of these

(viii) Mass energy equivalence relation is given by : 1

(a) $E = mc^2$
(b) $E = m/c^2$
(c) $E = c^2/m$
(d) None of these

(ix) The scalar potential ϕ and vector potential A of the electric field is : 1

(a) $\bar{E} = \text{grad}\,\phi - \frac{1}{c}\frac{\partial \bar{A}}{\partial t}$
(b) $\bar{E} = \text{grad}\,\phi + \frac{1}{c}\frac{\partial \bar{A}}{\partial t}$
(c) $\bar{E} = -\text{grad}\,\phi - \frac{1}{c}\frac{\partial \bar{A}}{\partial t}$
(d) $\bar{E} = -\text{grad}\,\phi + \frac{1}{c}\frac{\partial \bar{A}}{\partial t}$

(x) Four force f^i = ______ 1

(a) $\frac{du^i}{ds}$
(b) $\frac{dx^i}{ds}$
(c) $\frac{dp^i}{ds}$
(d) None of these

UNIT—I

2. (a) Define inertial system. Prove that in an inertial frame a body, without influence of any forces, moves in a straight line with constant velocity. 1+3

(b) Discuss the Geometrical interpretation of Lorentz transformation. 4

(c) Show that $x^2 + y^2 + z^2 - c^2t^2$ is Lorentz invariant. 2

3. (p) Prove that Newton's fundamental equations of motion are invariant under the Galilean transformation. 4

(q) What are the postulates of special relativity ? 2

(r) Show that the three dimensional volume element dxdydz is not Lorentz invariant but the four dimensional volume elements dxdydzdt is Lorentz invariant. 4

UNIT—II

4. (a) Derive the transformation for the acceleration of a particle. Prove that when u, v < < c, these transformation deduce to Galilean one. 6

(b) Obtain the relativistic transformation formulae for the velocities of particle. 4

5. (p) Obtain the transformation of the Lorentz contraction factor $\sqrt{1-\frac{u^2}{c^2}}$. 6

(q) An observer moving along the x-axis of S with velocity V observes a body of proper volume V_0 moving with velocity u along the x-axis of S. Show that the observer measures the volume to be equal to $V_0\sqrt{\frac{(c^2-v^2)(c^2-u^2)}{(c^2-uv)^2}}$. 4

UNIT—III

6. (a) Define four tensor.

Prove that :

(i) $T'^{11}=\alpha^2\left\{T^{11}-\frac{V}{C}T^{14}-\frac{V}{C}T^{41}+\frac{V^2}{C^2}T^{44}\right\}$

(ii) $T'^{12}=\alpha\left\{T^{12}-\frac{V}{C}T^{42}\right\}$ 1+2+2

(b) Define length of four radius vector. Show that $x^1=-x_1$, $x^2=-x_2$, $x^3=-x_3$, $x^4=x_4$ and then deduce that $x_i=(-\bar{r},\ ct)$. 1+3+1

7. (p) Define four vector A^r. Show that the square of the length of a four vector is invariant under Lorentz transformation. 1+4

(q) Prove that there exists an inertial system s' in which the two events occur at one and the same time if the interval between two events is space like. 5

UNIT—IV

8. (a) Prove that $L=-m_oc^2\sqrt{1-\frac{u^2}{c^2}}$ for the relativistic Lagrangian. 5

(b) Define four velocity. Prove that the four velocity, in component form can be expressed as :

$$u^i = \left(\frac{\bar{u}}{c \cdot \sqrt{1 - \frac{u^2}{c^2}}}, \frac{1}{\sqrt{1 - \frac{u^2}{c^2}}} \right),$$

where $\bar{u} = (u_x, u_y, u_z)$ = ordinary z-dimensional velocity of the particle. 5

9. (p) Obtain Einstein's mass energy equivalence relation. 5

(q) Define four velocity and four acceleration. Show that four velocity and four acceleration are mutually orthogonal. 2+3

UNIT—V

10. (a) Define electric and magnetic field strenths in terms of scalar ϕ and vector potential $\bar{A}$ and show that $\vec{E}$ and $\vec{H}$ remain invariant under Gauge transformation. 2+3

(b) Prove that the Lagrangian for a charge particle in electromagnetic field is :

$$L = -m_0 c^2 \sqrt{1 - \frac{u^2}{c^2}} + \frac{e}{c} \vec{A} \cdot \vec{u} - e\phi .$$ 5

11. (p) Show that the Hamiltonian for a charged particle moving in an electromagnetic field is :

$$H = \left\{ m_0^2 c^4 + c^2 \left(P - \frac{e}{c} A \right)^2 \right\}^{1/2} + e\phi .$$ 5

(q) State Maxwell's equations of electromagnetic theory in vacuum. Also find its equations in component form. 2+3