B.Sc. (Part—III) Semester-VI Examination

MATHEMATICS (OLD) (UPTO WINTER-2018)

(Linear Algebra)

Paper—XI

Time : Three Hours] [Maximum Marks : 60

Note :— (1) Question No. **1** is compulsory and attempt this question once only.

(2) Attempt **ONE** question from each unit.

1. Choose the correct alternative :

(i) For two subspaces U and W of V(F)

$V = U \oplus W \Leftrightarrow$ ______ 1

(a) $V = U + W$ and $U \cap W = \{0\}$ (b) $V = U + W$

(c) $U \cap W = \{0\}$ (d) None of these

(ii) If S is nonempty subset of vector space V, then L(S) is ______. 1

(a) Smallest subspace of V containing V

(b) Largest subspace of V containing S

(c) Smallest subspace of V containing S

(d) None of these

(iii) If $T : U \to V$ be a linear map and U be a finite dimensional vector space then dim R(T) + dim N(T) = ______ 1

(a) dim U (b) dim R

(c) dim N (d) None of these

(iv) The Kernel of a linear transformation $T : U \to V$ is a subspace of ______. 1

(a) U and V (b) U

(c) V (d) None of these

(v) If W is subspace of a vector space V over F, then $\{f \in \hat{V} / f(W) = 0, \forall w \in W\}$ is called as : 1

(a) Annihilation of W (b) Dual space of W

(c) Hilatory of W (d) None of these

(vi) If $\hat{V}$ is n-dimensional, then the dimension of V is : 1

(a) 0 (b) n

(c) Less than n (d) Greater than n

(vii) In an inner product space V, the inequality $|(u \cdot v)| \le \|u\| \cdot \|v\|$ for all $u, v \in V$ is known as : 1

(a) Triangle inequality (b) Bessel's inequality

(c) Schwartz inequality (d) None of these

(viii) If $\|V\| = 1$ then V is called : 1

(a) Standard inner product (b) Normalised

(c) Scalar inner product (d) Orthonormal

(ix) If A is any submodule of a R-module M, then the zero element of the quotient module $\frac{M}{A}$ is : 1

(a) A (b) M

(c) $\{0\}$ (d) None of these

(x) If V is IPS and $u, v \in V$ then u is said to be orthogonal to V if $(u, v) =$ ______. 1

(a) 1 (b) -1

(c) 0 (d) None of these

UNIT—I

2. (a) If S is a nonempty subset of a vector space V, then prove that L(S) is the smallest subspace of V containing S. 5

(b) Prove that a nonempty subset U of a vector space V(F) is a subspace of V iff $\alpha u + \beta v \in U \;\forall\; \alpha, \beta \in F$ and $\forall\; u, v \in U$. 5

3. (p) Let U and W be subspaces of a vector space V_3, where

$$U = \{(x_1, x_2, x_3) \in V_3 \mid x_3 = x_1 + x_2\},$$

$$W = \{(x_1, x_2, x_3) \in V_3 \mid x_1 = x_2 = x_3\}.$$

Show that V_3 is the direct sum of U and W. 5

(q) From given two LI vectors (1, 0, 1, 0), (0, −1, 1, 0) of V_4, find a basis of V_4 that includes these two vectors. 5

UNIT—II

4. (a) Let a mapping $T : V_2 \to V_2$ be defined by $T(x, y) = (x', y')$, where $x' = x\cos\theta - y\sin\theta$, $y' = x\sin\theta + y\cos\theta$. Show that T is a linear map. 5

(b) If $T : U \to V$ be a linear map then prove that :

(i) R(T) is a subspace of V

(ii) N(T) is a subspace of U. 5

5. (p) If $T : U \to V$ be a nonsingular linear map, then prove that $T^{-1} : V \to U$ is a linear 1-1 and onto map. 5

(q) Find the range, kernel, rank and nullity of the matrix $A = \begin{bmatrix} 3 & 1 & 2 \\ 2 & -1 & 0 \\ 1 & 1 & 1 \\ 0 & 1 & 2 \end{bmatrix}$ and verify Rank-Nullity theorem. 5

UNIT—III

6. (a) If W is a subspace of a finite dimensional vector space V, then prove that A(A(W)) = W. 5

(b) If W_1 and W_2 are subspaces of a finite dimensional vector space V, then show that $A(W_1 + W_2) = A(W_1) \cap A(W_2)$. 5

7. (p) Let U, V be finite dimensional complex vector spaces and $A : U \to V$, $B : U \to V$ be linear maps. If $\alpha \in C$, then prove that :

(i) $(A + B)^* = A^* + B^*$

(ii) $(\alpha A)^* = \bar{\alpha} A^*$. 5

(q) Let the linear maps $T : V_3 \to V_3$ and $S : V_3 \to V_3$ be defined as $T(x_1, x_2, x_3) = (2x_1 - 3x_2, x_1 + x_2, x_3)$, and $Se_1 = e_2 - e_3$, $Se_2 = e_3$, $Se_3 = e_1 + e_2 + e_3$. Determine the linear maps : (i) S + T, (ii) 2T. 5

UNIT—IV

8. (a) In an inner product space V, prove that $|(u \cdot v)| \leq \|u\| \|v\|, \forall u, v \in V$. 5

(b) Let V be an inner product space over F. In V define the distance d(u, v) from u to v by $d(u, v) = \|u - v\|$. Prove that :

(i) $d(u, v) = d(v, u)$

(ii) $d(u, v) \leq d(u, w) + d(w, v), \forall u, v, w \in V$. 2+3

9. (p) Using Gram-Schmidt orthogonalisation process, orthonormalise the set of vectors {(1, 0, 1, 0), (1, 1, 3, 0), (0, 2, 0, 1)} of V_4. 5

(q) If $W^\perp$ is the set of orthogonal vectors in an inner product space V, then prove that $W^\perp$ is a subspace of V. 5

UNIT—V

10. (a) Prove that every abelian group G is a module over the ring of integers Z. 5

(b) If M_1 and M_2 are submodules of R-module M, then $M_1 + M_2$ is a submodule of M. Moreover, $M_1 + M_2$ is a direct sum of M_1, $M_2 \Leftrightarrow M_1 \cap M_2 = \{0\}$. 5

11. (p) If T is a homomorphism of an R-module M to an R-module H, then prove that :

(i) $T_0 = 0$

(ii) $T(-m) = -Tm \ \forall m \in M$

(iii) $T(m_1 - m_2) = Tm_1 - Tm_2 \ \forall m_1, m_2 \in M$. 4

(q) If A and B are submodules of M, then prove that $\frac{A+B}{B} \cong \frac{A}{A \cap B}$. 6

http://www.sgbauonline.com/