AQ–740

B.Sc. (Part-III) Semester-VI Examination

MATHEMATICS—XI

(Linear Algebra)

Time—Three Hours] [Maximum Marks—60

Note :—(1) Question **ONE** is compulsory and attempt this question once only.

(2) Attempt **ONE** question from each unit.

1. Choose the correct alternative :

(1) Let V(F) be a vector space, M is a subspace of V if dim V = n, dim V/M = r, then dim M is ______. 1

(a) n/r

(b) n + r

(c) n – r

(d) r – n

(2) Which of the following is not vector space : 1

(a) R(R)

(b) C(R)

(c) R(C)

(d) C(C)

(3) If $T : U \to V$ is an on-to map, then _____. 1

(a) dim U < dim V

(b) dim U = dim V

(c) dimU/dim V

(d) dim U ⊕ dim V

(4) If $T : U \to V$ is the identity map, then Nullity = _____. 1

(a) 0

(b) 1

(c) 2

(d) 3

(5) If W_1 and W_2 are subspaces of vector space V(F) such that $W_1 \subset W_2$, then _____. 1

(a) $A(W_1) \subset A(W_2)$

(b) $A(W_2) \subset A(W_1)$

(c) $A(W_1) - A(W_2)$

(d) $A(W_1) + A(W_2)$

(6) If W is a subspace of finite dimensional vector space V(F), then dim W + dim (A(W)) is _____. 1

(a) 0

(b) 2

(c) dim W

(d) dim V

UNIT—V

10. (a) Let M be an R-module. Prove that the following : 3

(i) $r \cdot 0 = 0 \quad \forall\, r \in R$

(ii) $-(r \cdot a) = r(-a) = (-r)a \quad \forall\, r \in R,\ a \in M$

(iii) $r(a - b) = ra - rb \quad \forall\, r \in R,\ a, b \in M$.

(b) If M_1 and M_2 are submodules of R-module M, then prove that $M_1 + M_2$ is a submodule of M. Moreover $M_1 + M_2$ is a direct sum of M_1, M_2 iff $M_1 \cap M_2 = \{0\}$. 5

(c) Define : 2

(i) R-module homomorphism

(ii) Quotient module.

11. (p) If H and K are submodules of M, then prove that :

$$\frac{H+K}{K} \cong \frac{H}{H \cap K}.$$ 5

(q) If T is a homomorphism of a R-module M to an R-module N, then prove that : 3

(i) $T(0) = 0$

(ii) $T(-m) = -T(m), \ \forall\, m \in M$

(iii) $T(m_1 - m_2) = T(m_1) - T(m_2) \ \forall\, m_1, m_2 \in M$.

(r) Let M be an R-module and $m \in M$. Prove that $A = \{rm : r \in R\}$ is a submodule of M. 2

7. (p) If W is a subspace of finite dimensional vector space V, then prove that A(A(W)) = W. 4

(q) If K_λ is eigen space, then prove that K_λ is a subspace of vector space V. 3

(r) Show that A(W) is a subspace of dual space $\hat{V}$. 3

UNIT—IV

8. (a) If V is an IPS over F. If u, v ∈ V, then prove that :

$$\| u \circ v \| \leq \| u \| \cdot \| v \|.$$ 3

(b) Prove that every normal set is L.I. 3

(c) Apply Gram-Schmidt method to orthonormalise set {(1, 0, 1, 1), (–1, 0, –1, 1), (0, –1, 1, 1)}. 4

9. (p) Show that inner product can be defined on V_2,

by $$(x_1, x_2) \circ (y_1, y_2) = \frac{(x_1 - x_2)(y_1 - y_2)}{4} + \frac{(x_1 + x_2)(y_1 + y_2)}{4}.$$

In this inner product, Find :

(i) $e_1 \circ e_2$

(ii) (1, –1) ∘ (1, 1). 5

(q) Define IPS. If u and v are vectors in IPS, then show that : 1+4

(i) $\| u + v \|^2 + \| u - v \|^2 = 2(\| u \|^2 + \| v \|^2)$

(ii) $\| u + v \|^2 - \| u - v \|^2 = 2 \operatorname{Re}(u \circ v)$.

(7) The normalized vector of (1, –2, 5) is _____. 1

(a) $\left(\frac{1}{\sqrt{30}}, \frac{-2}{\sqrt{30}}, \frac{5}{\sqrt{30}}\right)$

(b) $\left(\frac{1}{2}, -1, \frac{5}{2}\right)$

(c) $\left(\frac{1}{5}, \frac{-2}{5}, 1\right)$

(d) (1, –2, 5)

(8) If W_1 and W_2 are subspace of IPS, V(F), then $(W_1 \cap W_2)^\perp$ = _____. 1

(a) $W_1^\perp \cap W_2^\perp$

(b) $W_1^\perp + W_2^\perp$

(c) $W_1^\perp \oplus W_2^\perp$

(d) $W_1^\perp - W_2^\perp$

(9) Let M and N be two R-modules and T : M → N be an R-homomorphism. If B is a submodule of N, then _____. 1

(a) $T^{-1}(B)$ is a submodule of M

(b) $T^{-1}(B)$ is a submodule of N

(c) $T^{-1}(B)$ is a kernel of R-homomorphism

(d) $T^{-1}(B) = T(M)$

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(10) The zero element of quotient module M/K is ______. 1

(a) M

(b) {0}

(c) K

(d) None of these

UNIT—I

2. (a) Define linear span. Let S be a non-empty subset of a vector space V. Then prove that (S) is smallest subspace of V containing S. 1+2

(b) Prove that the set {(1, 2, 1), (2, 1, 0), (1, –1, 2)} is a basis of V_3. 3

(c) Prove that :

(1, 0, 0), (0, 1, 0), (0, 0, 1) are L.I. and (1, 1, 0), (3, 1, 3), (5, 3, 3) are L.D. over F. 4

3. (p) Define subspace of vector space. Prove that intersection of two subspaces of a vector space is a subspace. 1+2

(q) If x, y, z are LI vectors of a vector space V then prove that x+y, y+z, z+x are LI. 3

(r) Let S = [(0, 1, 0), (0, 0, 1)] and T = [(1, 2, 0), (3, 1, 2)] be subspace of V_3. Find basis and dim of $S \cap T$ and S + T. 4

UNIT—II

4. (a) Define L.T. Find a Linear Transform $T : V_2 \to V_2$ such that T(1, 0) = (1, 1), T(0, 1) = (–1, 2). Prove that T maps a square with vertices (0, 0), (1, 0), (1, 1) and (0, 1) in a parallelogram. 1+3

(b) Let $T : U \to V$ be a linear map. Show that if T is one-one and $u_1, u_2, u_3 \ldots u_n$ are L.I. vectors in U, then $T_{u_1}, T_{u_2}, T_{u_3}, \ldots T_{u_n}$ are L.I. vectors in V. 4

(c) Let $T : V_2 \to V_2$ be a linear map, defined by $T(x_1, x_2) = (2x_1 + 3x_2, x_1 - x_2)$. Show that T is one-one and onto. 2

5. (p) If matrix of linear map T, with respect to basis B_1 and B_2 is $\begin{bmatrix} 1 & 1 & 2 \\ 2 & 3 & 1 \end{bmatrix}$, where B_1 = {(1, 1), (1, 0)} and B_2 = {(1, 1, 0), (0, 1, 1), (1, 0, 1)}. Find T(x, y, z). 5

(q) State and prove rank-nullity theorem. 5

UNIT—III

6. (a) Prove that eigen vectors corresponding to distinct eigen values of square matrix are L.I. 5

(b) If V is the finite dimensional vector space over F, then prove that $V \cong \hat{V}$. 5