AU–157

B.Sc. (Part-III) Semester-VI Examination
MATHEMATICS (New)
(Linear Algebra)
Paper—XI

Time : Three Hours] [Maximum Marks : 60

Note :— (1) Question No. **1** is compulsory and attempt it once only.

(2) Attempt **ONE** question from each unit.

1. Choose the correct alternatives : 10

(i) The basis {(1, 0, 0), (0, 1, 0), (0, 0, 1)} of the vector space $R^3(R)$ is known as :

(a) Normal basis (b) Quotient basis

(c) Standard basis (d) None of these

(ii) The vectors (a, b) and (c, a) are L.D iff :

(a) ad – bc = 0 (b) ab – cd = 0

(c) cd – ab = 0 (d) ab + dc = 0

(iii) The kernel of a linear transformation $T : U \to V$ is a subset of :

(a) U (b) V

(c) U and V (d) None of these

(iv) If W is a subspace of a finite dimensional vector space V, then dim (V/W) =

(a) $\frac{\dim V}{\dim W}$ (b) dim V – dim W

(c) dim V + dim W (d) None of these

(v) An element of dual space of V is called a :

(a) Linear functional (b) Bilinear element

(c) Linear element (d) None of these

(vi) Annihilator of W, A(W) is a subspace of :

(a) W (b) V

(c) $\hat{V}$ (d) None of these

(vii) Every set of orthogonal vectors is :

(a) Linearly Independent

(b) Linearly Dependent

(c) Linearly Independent and Linearly Dependent

(d) None of these

(viii) Let W be a subspace of an IPSV then $W \cap W^{\perp} =$

(a) $\{0\}$ (b) $\{1\}$

(c) ϕ (d) None of these

(ix) R-Module homomorphism is linear transformation if :

(a) R is with unit element (b) R is commutative

(c) R is a field (d) None of these

(x) If the ring R has a unit element 1 and 1.a = a for all $a \in M$, then M is called :

(a) A unital R-module (b) Right R-module

(c) Left R-module (d) None of these

UNIT—I

2. (a) Prove that intersection of two subspaces of a vector space is again a subspace. Is this statement is true for union ? 5

(b) Let U and W are two subspaces of a vector space V and Z = U + W. Then show that $Z = U \oplus W \Leftrightarrow z = u + w$ uniquely for any $z \in Z$ and for some $u \in U$ and $w \in W$. 5

3. (p) Define the Linear span of a subset of a vector space and show that Linear span L(S) of a subset S of a vector space V is the smallest subspace of V containing S. 5

(q) If U and W are finite dimensional subspaces of a vector space V, then prove that :

$$\dim(U + W) = \dim U + \dim W - \dim (U \cap W).$$ 5

UNIT—II

4. (a) Let $T : U \rightarrow V$ be a linear transformation. Then prove that :

T is one-one $\Leftrightarrow$ $N(T)$ is zero subspace of U. 5

(b) Let $T : V_3 \rightarrow V_3$ defined by :

$T(x_1, x_2, x_3) = (x_1 + x_2, x_2 + x_3, x_3 - 2x_1)$

Find range, kernel, rank, nullity and verify rank-nullity theorem. 5

5. (p) State and prove Rank-Nullity theorem. 5

(q) Find the transformation $T(x, y, z)$. If T is a linear map and matrix of T with respect to the bases B_1 and B_2 is $\begin{bmatrix} -1 & 2 & 1 \\ 1 & 0 & 3 \end{bmatrix}$, where

$B_1 = \{(1, 2, 0), (0, -1, 0), (1, -1, 1)\}$ and

$B_2 = \{(1, 0), (2, -1)\}$. 5

UNIT—III

6. (a) Let U, V are finite dimensional complex vector spaces and $A : U \rightarrow V$, $B : U \rightarrow V$ be linear maps, $\alpha \in C$, then prove that :

(i) $(A + B)^* = A^* + B^*$

(ii) $(\alpha A)^* = \bar{\alpha} A^*$ 3+2

(b) Prove that the element $\lambda \in f$ is a characteristic root of $T \in L(V)$ iff for some non zero $v \in V$, $Tv = \lambda v$. Also define characteristic root and characteristic vector. 3+1+1

7. (p) If V is a finite dimensional vector space over F, then prove that $V \approx \hat{V}$. 5

(q) If W is a subspace of finite dimensional vector space V, then prove that $A(A(W)) = W$. 5

UNIT—IV

8. (a) Define inner product space and prove that in an inner product space V :

(i) $\| \alpha \cdot u \| = | \alpha | \cdot \| u \|$

(ii) $\| u + v \| \leq \| u \| + \| v \|$, $\alpha \in F$ and $u, v \in V$. 1+4

(b) Using Gram-Schmidt process orthonormalise the set of vectors $\{(1, 0, 1, 0), (1, 1, 3, 0), (0, 2, 0, 1)\}$ of V_4. 5

9. (p) Prove that if $\{W_1, W_2, \ldots, W_m\}$ is an orthonormal set in V, then $\sum_{i=1}^{m} |(w_i, v)|^2 \leq \| v \|^2$ for any $v \in V$. 5

(q) Prove that every finite dimensional inner product space has an orthogonal basis. 5

UNIT—V

10. (a) Define Homomorphism of Modules and prove that if T is a homomorphism of an R-module M to an R-Module H, then :

(i) $T(0) = 0$

(ii) $T(-m) = -T(m) \ \forall \ m \in M$

(iii) $T(m_1 - m_2) = T(m_1) - T(m_2) \ \forall \ m_1 m_2 \in M$. 1+4

(b) Prove that every abelian group G is a module over a ring of integers Z. 5

11. (p) Define the sub module and prove that an arbitrary intersection of sub modules of a module is a submodule. 1+4

(q) Define direct sum of submodules and prove that if M_1 and M_2 are sub modules of R-module M, then $M_1 + M_2$ is a submodule of M. 1+4