# B.Sc. Part-III (Semester-VI) Examination

## MATHEMATICS

## (Graph Theory)

## Paper—XII

Time : Three Hours] [Maximum Marks : 60

**Note :—** (1) Question No. **1** is compulsory and attempt it at once only.

(2) Solve **ONE** question from each unit.

1. Choose the correct alternative in the following :

(i) A connected graph G is an Euler graph iff it can be decomposed into : 1

(a) Walks (b) Paths

(c) Cut sets (d) Circuits

(ii) A subgraph $H = < V_1, E_1 >$ of a graph $G = < V, E >$ is called a spanning subgraph if : 1

(a) $E_1 = \phi$ (b) $V_1 = \phi$

(c) $V_1 = V$ (d) $E_1 = E$

(iii) The concept of a tree was introduced by : 1

(a) Euler (b) Hamiltonian

(c) Cayley (d) Kuratowski

(iv) If G be a circuitless graph with n vertices and k components then G has : 1

(a) n + 1 edges (b) n – 1 edges

(c) n + k edges (d) n – k edges

(v) A graph can be embedded in the surface of a sphere iff it can be embedded in : 1

(a) a plane (b) a circle

(c) a sphere (d) a straight line

(vi) A complete graph of five vertices is : 1

(a) Planar graph (b) Non-planar graph

(c) Null graph (d) Bipartite graph

(vii) Minimum number of linearly independent vectors that spans the vectors in a vector space $W_G$ is called : 1

(a) Basis of vector space (b) Dimension of vector space

(c) Span (d) None of these

(viii) The dimension of the cutspace $W_S$ is equal to the rank of the graph and the number of cutset vectors including 0 in $W_S$ is : 1

(a) r (b) $2^r$

(c) $3^r$ (d) $r^2$

(ix) A row with all zeros in incidence matrix represents : 1

(a) Pendent vertex (b) Isolated vertex

(c) Odd vertex (d) Even vertex

(x) If B is a circuit matrix of a connected graph G with n vertices and e edges then rank of B is : 1

(a) e + n – 1 (b) e – n – 1

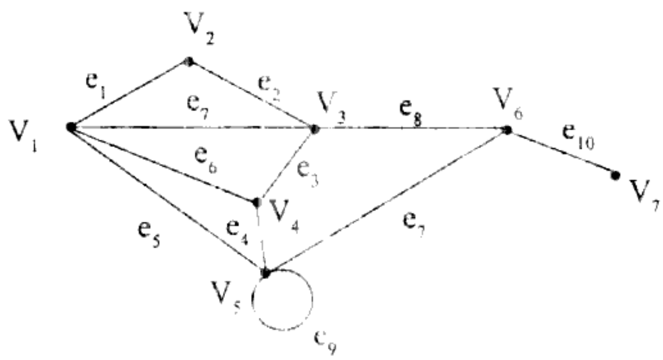
(c) e + n + 1 (d) e – n + 1

## UNIT—I

2. (a) Define (i) Simple graph, (ii) Degree of a vertex. Show that the maximum number of edges in a simple graph of n vertices is $\frac{n(n-1)}{2}$. 2+3

(b) Define isomorphism between two graphs. Prove that any two simple connected graphs with n vertices, all of degree two are isomorphic. 2+3

3. (p) From the graph given below answer the following :




(i) Write the degree of each vertex.

(ii) Which edges are incident with the vertex $V_3$ ?

(iii) Write the adjacent vertices of $V_5$.

(iv) Is the graph simple ? Why ? 1+1+1+2

(q) In a graph G there exists a path from the vertex u to the vertex v iff there exists a walk from u to v. 5

## UNIT—II

4. (a) Prove that following statements are equivalent :

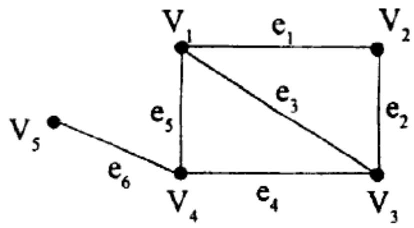
(i) There is exactly one path between every pair of vertices in G.

(ii) G is minimally connected graph. 5

(b) Define : (i) Binary tree, (ii) Rooted tree. Show that there are (n + 1)/2 number of pendent vertices in a binary tree with n vertices. 2+3

5. (p) Define eccentricity of a vertex. Show that every tree has either one or two centres. 1+4

(q) Define spanning tree and find out all possible spanning trees of the following graph. 1+4

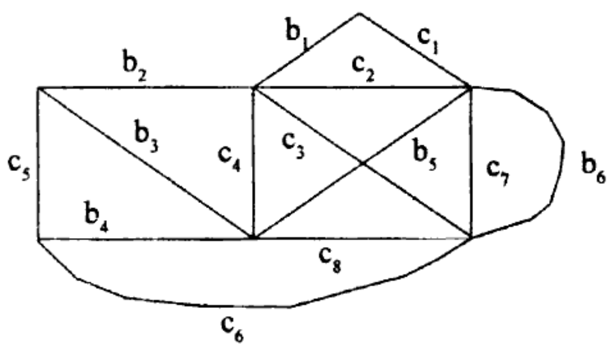



## UNIT—III

6. (a) Define planar graph. If G is planar graph with n vertices, e edges, f faces and k components then prove that $n - e + f = k + 1$. 1+4

(b) Prove that every cutset in a connected graph G must contain at least one branch of every spanning tree of a graph G. 5

7. (p) Define fundamental circuits for the following graph G, find rank of G, nullity of G and fundamental circuits with reference to the spanning tree : $T = \{b_1, b_2, b_3, b_4, b_5, b_6\}$. 1+4



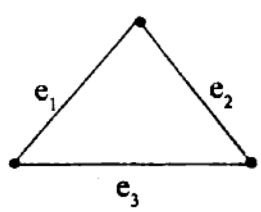

**Graph G**

(q) Show that Kuratowski's $K_{3,3}$ graph is non-planar. 5

## UNIT—IV

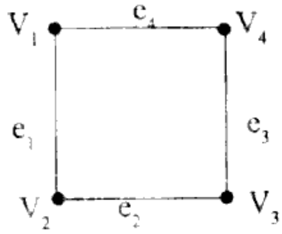
8. (a) Prove that in the vector space of a graph the circuit subspace and cutset subspace are orthogonal to each other. 5

(b) For a graph G with spanning tree $T = \{e_1, e_2\}$ find $W_G$, $W_S$, $W_\Gamma$, $W_\Gamma \cap W_S$ and $W_\Gamma \cup W_S$. 5




9. (p) Prove that the set $W_\Gamma$ of all circuit vectors including zero vector in $W_G$ form a subspace of $W_G$. 5

(q) Let G be a graph given as in figure. Find $W_\Gamma$, $W_S$, $W_\Gamma \cap W_S$ and $W_\Gamma \cap W_S$ where $W_\Gamma$ is a circuit subspace and $W_S$ is a cutset subspace. 5

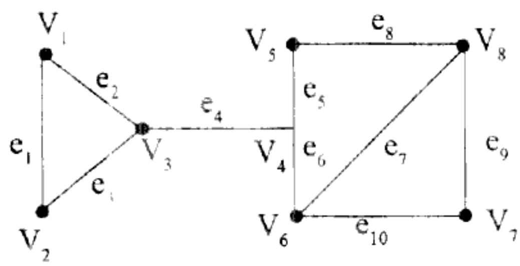



Group G

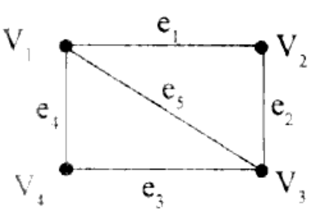
## UNIT—V

10\. (a) Prove that the reduced incidence matrix of graph is non-singular iff the graph is a tree. 5

(b) Define circuit matrix. Find the circuit matrix of the graph. 1+4




11\. (p) Find incidence matrix A(G), circuit matrix B(G) and show that $AB^T = 0$, for the following graph. 5




**Graph G**

(q) Define the Adjacency matrix. Find the Adjacency matrix of the following graph. 1+4