AU–154

B.Sc. (Part-III) Semester-VI Examination
MATHEMATICS (OLD) UPTO WINTER-2018
(Graph Theory) (Optional)
Paper—XII

Time : Three Hours] [Maximum Marks : 60

Note :—(1) Question No. 1 is compulsory and attempt it at once.

(2) Attempt **ONE** question from each unit.

1. Choose the correct alternative :

(i) If a pair of vertices is associated with more than one edge then edges are called : 1

(a) Self loop (b) Parallel edges

(c) Incident edge (d) None of these

(ii) A vertex of degree one is called : 1

(a) isolated vertex (b) end vertex

(c) terminal vertex (d) pendent vertex

(iii) A connected graph without any circuits is called : 1

(a) cut set (b) complete graph

(c) tree (d) None of these

(iv) Every connected graph has at least ______ spanning tree. 1

(a) 1 (b) 2

(c) 3 (d) 4

(v) The minimum number of vertices whose removal from connected graph leaves the remaining graph disconnected is called : 1

(a) edge connectivity (b) separability

(c) vertex connectivity (d) None of these

(vi) Every cut-set in a non-separable graph with more than two vertices contains at least : 1

(a) one edge (b) two edges

(c) three edges (d) None of these

(vii) The dimension of the cut-set subspace W_S is equal to the rank of the graph and the number of cut-set vectors (including 0) in W_S is : 1

(a) 3^r (b) 2^r

(c) r (d) None of these

(viii) The dot product of two vectors, one corresponding a subgraph g and the other g' is _____ if the number of edges common to g and g' is even. 1

(a) one (b) two

(c) three (d) zero

(ix) If A(G) is an incidence matrix of a connected graph G with n vertices then the rank of A(G) is : 1

(a) n (b) n – 1

(c) n – 2 (d) None of these

(x) In a path matrix there is no row with all : 1

(a) Zeros (b) Ones

(c) Vertices (d) Edges

UNIT—I

2. (a) Define Graph. If a graph (connected or disconnected) has exactly two vertices of odd degree, then prove that there must be a path joining these two vertices. 1+4

(b) Prove that a connected graph G is a Euler graph if and only if it can be decomposed into circuits. 5

3. (p) Define simple graph. Prove that a simple graph with n vertices and k components can have at most (n – k) (n – k + 1)/2 edges. 1+4

(q) Define even and odd vertices and show that in a connected graph there are even number of odd degree vertices. 5

UNIT—II

4. (a) Prove that a graph G with n vertices, n – 1 edges and no circuits is connected. 5

(b) Prove that every connected graph has at least one spanning tree. 5

5. (p) Define distance between two vertices in connected graph. Prove that the distance between two vertices in connected graph is a metric. 1+4

(q) Define tree. Prove that any connected graph with n vertices and n – 1 edges is a tree. 1+4

UNIT—III

6. (a) Prove that every circuit has an even number of edges in common with any cut-set. 5

(b) Prove that the complete graph of five vertices is non-planar. 5

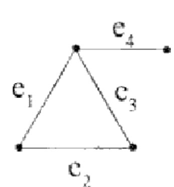
7. (p) Prove that the ring sum of any two cut-sets in a graph is either a third cut-set or an edge disjoint union of cut-sets. 5

(q) Prove that a connected planar graph with n vertices and edges has e – n + 2 regions. 5

UNIT—IV

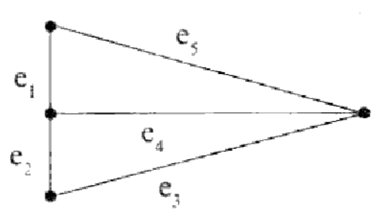
8. (a) Prove that the set of circuit vectors corresponding to the set of fundamental circuit, with respect to any spanning tree, forms a basis for the circuit subspace W_Γ. 5

(b) Find W_Γ, W_S, $W_\Gamma \cap W_S$ and $W_\Gamma \cup W_S$ for the following graph. 5



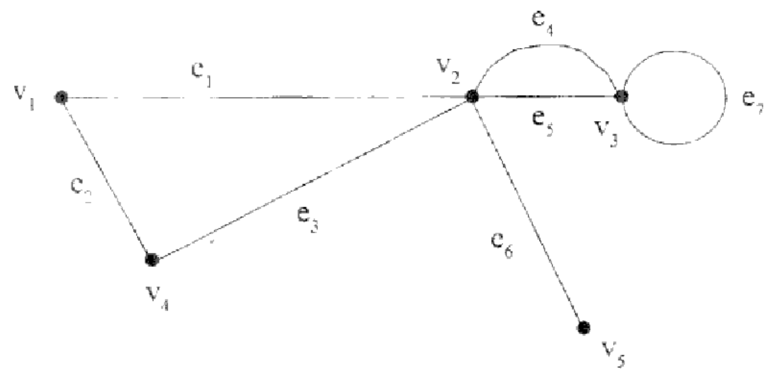
9. (p) Prove that the circuit space W_Γ and the cutset subspace W_s are orthogonal to each other in the vector space of a graph. 5

(q) Find all circuits and cutsets of the graph G given below and calculate W_s and W_Γ and their dimensions. 5



UNIT—V

10. (a) Define incidence matrix. Find incidence matrix A(G) for the following graph. 1+4



(b) Let A and B be respectively, the incidence matrix and the circuit matrix of a loop free graph whose columns are arranged using the same order of edges. Then show that every row of A is orthogonal to every row of B. i.e., $A \cdot B^T = 0$, $B \cdot A^T = 0$ (mod² 2), where superscript T denotes the transposed matrix. 5

11. (p) If B is a circuit matrix of a connected graph G with e edges and n vertices then prove that rank of $B = e - n + 1$. 5

(q) Define fundamental circuit matrix and find fundamental circuit matrix (with respect to the spanning tree shown in heavy lines) of the following graph. 5