B.Sc. (Part-III) Semester-VI Examination

MATHEMATICS (NEW)

(Graph Theory) (Optional)

Paper—XII

Time : Three Hours] [Maximum Marks : 60

Note :— (1) Question No. **1** is compulsory and attempt it at once only.

(2) Solve **ONE** question from each unit.

1. Choose the correct alternative in the following :

(i) The vertex with degree one is called as :

(a) even vertex (b) odd vertex

(c) pendant vertex (d) isolated vertex

(ii) For any graph G with e edges and n vertices, sum of degree of all vertices is equal to :

(a) 2e (b) $\frac{e}{2}$

(c) e + 1 (d) e – 1

(iii) Every connected graph has at least :

(a) one spanning tree (b) two spanning trees

(c) three spanning trees (d) None of these

(iv) The total number of pendant vertices in a binary tree with n vertices are :

(a) n – 1 (b) n + 1

(c) $\frac{n-1}{2}$ (d) $\frac{n+1}{2}$

(v) A complete graph of four vertices is :

(a) non-planar graph (b) planar graph

(c) regular graph (d) complete graph

(vi) For any connected graph with n vertices, e edges its spanning tree has :

(a) $e - n - 1$ chords (b) $e - n + 1$ chords

(c) $e + n + 1$ chords (d) $e - n + 2$ chords

(vii) Two subspaces W_1 and W_2 are said to be orthogonal to each other iff $X \cdot Y =$ ____ (for all $X \in W_1$ and $Y \in W_2$)

(a) 0 (b) 1

(c) $X - Y$ (d) $X + Y$

(viii) If no vertex appears more than once in an edge sequence then it is called :

(a) a path (b) a walk

(c) a circuit (d) a cutset

(ix) In a graph without loops, the entries along principal diagonal of adjacency matrix are all :

(a) Zeros (b) Ones

(c) Purely imaginary (d) Complex number

(x) there is no rows with all zeros in the :

(a) incidence matrix (b) circuit matrix

(c) cutset matrix (d) path matrix $10\times1=10$

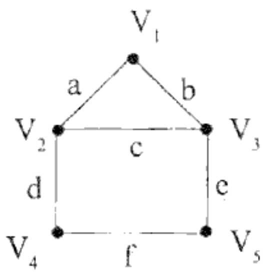
UNIT—I

2. (a) Define a simple graph. Show that the maximum number of edges in a simple graph of n vertices is $\frac{1}{2}n(n-1)$. 1+4

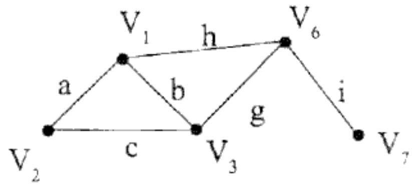
(b) Prove that a connected graph G is an Euler graph if and only if it can be decomposed into circuits. 5

3. (p) Define degree of a vertex. Prove that there are even number of odd degree vertices. 1+4

(q) Define union and ring sum of two graphs G_1 and G_2. Graphs G_1 and G_2 are shown below. Find (i) $G_1 \cup G_2$, (ii) $G_1 \cap G_2$, (iii) $G_1 \oplus G_2$. 1+1+1+1+1



Graph G_1



Graph G_2

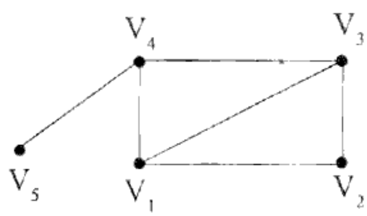
UNIT—II

4. (a) Define tree. Prove that a graph is a tree if and only if it is minimally connected. 1+4

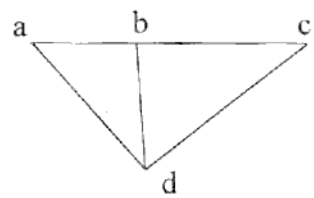
(b) Define eccentricity of a vertex. Show that every tree has either one or two centres. 1+4

5. (p) If T_1 and T_2 are two spanning trees of a connected graph G and if edge e is in T_1 but not in T_2, then prove that there exists another edge f in T_2 but not in T_1 such that subgraph $(T_1 - f) \cup f$ and $(T_2 - f) \cup e$ are also spanning trees of G. 5

(q) Sketch all spanning trees of the following graphs G_1 and G_2. 3+2



Graph G_1



Graph G_2

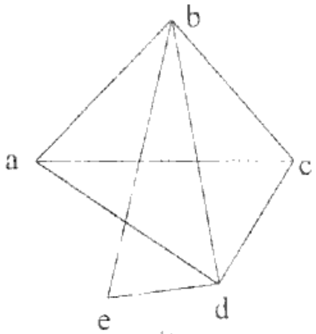
UNIT—III

6. (a) Using geometric arguments, prove that the graph $K_{3,3}$ is non planar. 5

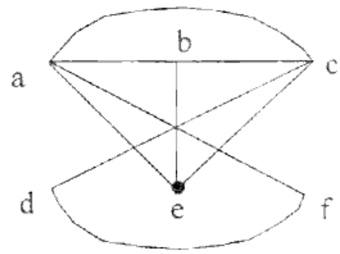
(b) If G is planar graph with n vertices, e edges, f faces and k components then prove that $n - e + f = k + 1$. 5

7. (p) Define cut set. Prove that every cut set in a connected graph G must contain at least one branch of every spanning tree of a graph G. 1+4

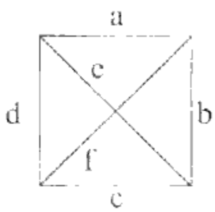
(q) Show that each graph is planar by redrawing it such that no edges cross. 2+2+1



(i)



(ii)



(iii)

UNIT—IV

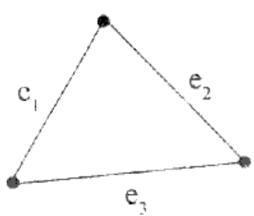
8. (a) Let G be a graph and W_G = set of all subgraphs of the graph G. Prove that W_G is vector space over the field Z_2 where the vector sum is the ring sum and scalar multiplication is given by :

$$\alpha \cdot X = \begin{cases} X \text{ if } \alpha = 1 \\ \phi \text{ if } \alpha = 0 \end{cases}$$

where $x \in W_G$ and $\alpha \in Z_2$, ϕ = null graph. 5

(b) Prove that set of all cut-set vectors in W_G forms a subspace W_S. 5

9. (p) For a graph G with spanning tree $T = \{e_1, e_2\}$ given by following figure, find W_G, W_S, W_Γ, $W_\Gamma \cap W_S$ and $W_\Gamma \cup W_S$. 5

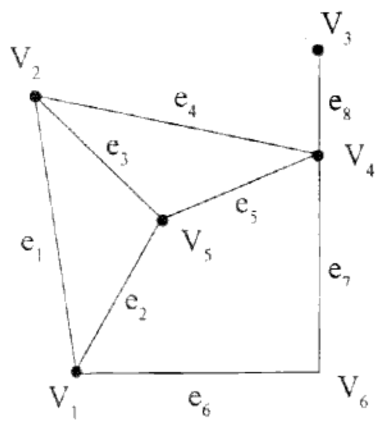


(q) Prove that the set of circuit vectors corresponding to the set of fundamental circuits, with respect to any spanning tree, forms a basis for the circuit subspace W_Γ. 5

UNIT—V

10. (a) Prove that the reduced incidence matrix of a graph is non-singular iff the graph is a tree. 5

(b) Define adjacency matrix. Find the adjacency matrix of the graph G. 1+4



11. (p) Prove that two graphs G_1 and G_2 are isomorphic iff their incidence matrices $A(G_1)$ and $A(G_2)$ are identical or differ only by permutations of rows and columns. 5

(q) Define circuit matrix. Find the circuit matrix of the graph. 1+4