AT–374

B.Sc. (Part-III) Semester-V Examination

MATHEMATICS (OLD) (UPTO SUMMER-2018)

Modern Algebra

Paper—X

Time : Three Hours] [Maximum Marks : 60

Note :— Question No. **1** is compulsory and attempt it once and solve **ONE** question from each unit.

1. Choose the correct alternative (1 mark each) :

(i) A subgroup H of a group G is a normal subgroup of G iff :

(a) $Hg = H$, for all $g \in G$ (b) $gH = H$, for all $g \in G$

(c) $Hg = gH$, for all $g \in G$ (d) $Hg = gH$, for some $g \in G$ 1

(ii) Let $(G, +)$ be a group. Then mapping $\phi : G \to G$ is homomorphism if :

(a) $\phi(a + b) = \phi(a) + \phi(b)$ (b) $\phi(a \cdot b) = \phi(a) \cdot \phi(b)$

(c) $\phi(a - b) = \phi(a) - \phi(b)$ (d) $\phi(a/b) = \phi(a)/\phi(b)$ 1

(iii) A group having no proper normal subgroup is called :

(a) a permutation group (b) a simple group

(c) a finite group (d) None of these 1

(iv) If f be a homomorphism of group G onto G' with Kernel K, then G' is :

(a) isomorphic to G/K (b) isomorphic to K/G

(c) isomorphic to G (d) isomorphic to G'/K 1

(v) A ring $(M, +, \cdot)$ of all 2×2 matrices over reals is :

(a) a commutative ring (b) a ring with zero divisors

(c) a ring without unity (d) None of these 1

(vi) The characteristic of a finite integral domain is :

(a) even number (b) odd number

(c) prime number (d) None of these 1

(vii) Which of the following polynomial is monic ?

(a) $(2x^2 + x - 1) \cdot (x^2 + 1)$
(b) $(2x^2 + x + 1) \cdot \left(\frac{1}{2}x^2 - x - 1\right)$
(c) $(2x^2 + x + 1) \cdot (-x - 1)$
(d) $(2x^2 + x + 1) \cdot (x^2 - 1)$ 1

(viii) A commutative ring which has no zero divisors is called :

(a) Boolean ring
(b) Integral domain
(c) Division ring
(d) None of these 1

(ix) the minimum number of element in any field is :

(a) 1
(b) 2
(c) 3
(d) None of these 1

(x) The polynomial $1 + x + x^3 + x^4$ is :

(a) irreducible over rationals
(b) irreducible over complex field
(c) not irreducible over any field
(d) None of these 1

UNIT—I

2\. (a) Prove that a subgroup N of G is a normal subgroup of G if and only if the product of two right coset of N in G is again a right coset of N in G. 4

(b) Show that the intersection of two normal subgroups of group G is a normal subgroup of G. 4

(c) Show that every subgroup of an abelian group is normal. 2

3\. (d) Prove that N is a normal subgroup of group G if and only if $gNg^{-1} = N \; \forall \; g \in G$ i.e. $Ng = gN \; \forall \; g \in G$. 4

(e) If $G = \{1, -1, i, -i\}$ and $N = \{1, -1\}$, then show that N is a normal subgroup of the multiplicative group G. Find the quotient group G/N and find its identity. 4

(f) Show that if G is abelian, then the quotient group G/N is also abelian. 2

UNIT—II

4\. (a) If G be any group and g a fixed element in G. The mapping $\phi : G \to G$ defined by $\phi(x) = gxg^{-1}$, then prove that ϕ is an isomorphism of G onto G. 5

(b) Define Homomorphism. If ϕ is an homomorphism of a group G into a group G, then prove that :

(i) $\phi(e) = e'$

(ii) $\phi(x^{-1}) = (\phi(x))^{-1} \forall x \in G$

where e and e' are identities of G and G' respectively. 5

5. (c) If M, N are normal subgroup of G, then prove that :

$$\frac{NM}{M} \approx \frac{N}{N \cap M}.$$ 5

(d) Prove that any in finite cyclic group is isomorphic to the additive group of integers. 5

UNIT—III

6. (a) Define subring. Prove that an arbitrary intersection of subring of a ring is a subring. 1+4

(b) Define a ring with zero divisors. Prove that a ring R is without zero divisors iff cancellation laws hold in R. 1+4

7. (c) Show that the set M of 2 × 2 matrices of the form $\begin{bmatrix} a & o \\ b & c \end{bmatrix}$; is a subring of ring of 2 × 2 matrices $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ with respect to the operation addition and multiplication of the matrices; where a, b, c, d are the integers. 5

(d) If in a ring R, $x^3 = x, \forall x \in R$, then show that R is commutative. 5

UNIT—IV

8. (a) Prove that every finite integral domain is a field. 5

(b) Prove that the characteristic of an integral domain is either zero or a prime number. 5

9. (c) Prove that every prime field of characteristic zero is isomorphic to the field Q of rational numbers. 5

(d) If R is a ring in which $x^2 = x \forall x \in R$, then prove that R is a commutative ring of characteristic 2. 5

UNIT—V

10. (a) If the primitive polynomial $f(x)$ can be factored as the product of two polynomials having rational coefficients, then prove that it can be factored as the product of two polynomials having integer coefficients. 5

(b) If $f(x)$ is a polynomial over a field F and $\alpha \in F$, then show that $f(\alpha)$ is the remainder when $f(x)$ is divided by $(x - \alpha)$. 3

(c) State Division Algorithm theorem for polynomials over a field F. 2

11. (d) Prove that R is an integral domain iff $R[x]$ is an integral domain. 5

(e) Prove that the polynomial $f(x) = x^4 - 2x + 2$ is irreducible over the field of rational numbers. 3

(f) Find the quotient and remainder upon dividing $f(x) = 6x^4 + x^3 + 6x^2 - 4x - 2$ by $g(x) = 2x^2 - x + 1$, where $f(x), g(x) \in Z_7[x]$. 2

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