11. (c) Prove that the polynomial $1 + x + x^2 + \cdots + x^{p-1}$; where P is a prime number; is irreducible over the field of rational numbers. 5

(d) Let p(x) be a prime polynomial in F[x]. If $p(x) \mid f(x).g(x)$, then prove that $p(x) \mid f(x)$ or $p(x) \mid g(x)$, where f(x) and g(x) are polynomials over a field f. 3

(e) Show that the polynomial x^2-3 is irreducible over the field of rational numbers. 2

Fifth Semester B. Sc. (Part - III) Examination

MATHEMATICS – X

(Modern Algebra)

P. Pages : 8

Time : Three Hours] [Max. Marks : 60

Note : Question **one** is compulsory and attempt it once and solve **one** question from each unit.

1. Choose the correct alternative (1 mark each):—

(1) If N is a normal subgroup of finite group G then 0(G/N) is equal to :

(a) 0(G) · 0(N) (b) 0(G)/0(N)

(c) 0(G) – 0(N) (d) 0(G) + 0(N)

(2) If N is normal subgroup of group G, then group G/N is known as :

(a) Factor group (b) Improper group

(c) Cyclic group (d) Proper group

(3) Kernel of homomorphism of group is a subgroup of :

(a) Domain set.

(b) Co–domain set

(c) Intersection of domain and codomain

(d) All the above.

(4) If $\phi : G \longrightarrow G'$ be an isomorphism, then for each $a \in G$

(a) $0(a) = 0(G(0))$

(b) $0(a) = 0(\phi(a))$

(c) $0(a) = 0\ (f(a))$

(d) $0(a) = 0(e)$.

(5) Which of the following is not an integral domain ?

(a) $(Q, +, \cdot)$ (b) $(R, +, \cdot)$

(c) $(C, +, \cdot)$ (d) $(N, + \cdot)$

(6) If $(f(x), g(x)) = d(x)$, then which one of the following is not true :

(a) $d(x)$ is monic

(b) $d(x)|f(x)$ and $d(x)|g(x)$

(c) $d'(x)|f(x)$ and $d'(x)|g(x) \Rightarrow d'(x)|d(x)$

(d) $d'(x)|f(x)$ and $d'(x)|g(x) \Rightarrow d(x) \mid d'(x)$.

UNIT IV

8. (a) Prove that every finite integral domain is a field. 5

(b) If R is a ring in which $x^2 = x$; $\forall x \in R$, then prove that R is a commutative ring of characteristic 2. 5

9. (c) Prove that every field is an integral domain. Does its converse true ? Justify. 5

(d) Prove that any two isomorphic integral domain have isomorphic quotient field. 5

UNIT V

10. (a) If f is a field, $\alpha \in F$ and $f(x) \in F[x]$, then prove that $(x-\alpha)$ is a factor of $f(x)$ iff $f(\alpha) = 0$. 4

(b) Let $f(x) = a_0 + a_1x + \text{------} + a_nx^n$ be a polynomial with integer coefficients. Suppose that for some prime number P ; $P \nmid a_n$, $P|a_1$, $P|a_2$, ------ $P|a_{n-1}$; $P|a_0$, $P^2 \nmid a_0$. Then prove that $f(x)$ is irreducible over the rationals. 6

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(f) Let G be a group of real numbers with respect to addition and $\phi: G \to G$ such that $\phi(x) = 13x \ \forall x \in G$; then prove that ϕ is homomorphism and hence find its kernel. 2

UNIT III

6. (a) Show that the set 'R' of integers modulo 7 under the addition and multiplication modulo 7 is a commutative ring with unity. 5

(b) Prove that 'S' subring of ring R iff $a+(-b) \in S$ and $a \cdot b \in S$; for all $a, b \in S$. 5

7. (c) Define ring without zero divisor. If R is a ring with unity, then prove that :

(i) $a.(b-c) = a.b - a.c$

(ii) $(-a) \cdot (-b) = a \cdot b$

(iii) $(-1) \cdot a = -a$. ; $\forall a, b, c \in R$. 1+4

(d) If Z is the set of integers on which two operations defined as $a \oplus b = a+b-1$ and $a \odot b = a+b-ab$; for every $a, b \in z$; then prove that$(z ; \oplus, \odot)$ is a commutative ring with unity. 5

(7) The identity element of the quotient group $G|H$ is :

(a) H (b) G

(c) $G|H$ (d) e.

(8) If $f: R \longrightarrow R^+$ defined by $f(x) = e^x$, then which of the following is not true :

(a) f is homomorphism

(b) f is not homomorphism

(c) f is isomorphism

(d) f is one to one.

(9) In ring R if $a^2 = a$ for all $a \in R$ then R is called.

(a) Division ring

(b) Boolean ring

(c) Polynomial ring

(d) Ring of quotient.

(10) If $f(x) = (a_0, a_1, a_2)$ be a polynomial over the field of reals and $\alpha + i\beta$ be the zero of f(x), then other zero of f(x) is :

(a) α (b) β

(c) $\alpha - i\beta$ (d) None of these.

10

UNIT I

2. (a) Prove that every subgroup of cyclic group is a normal subgroup. 3

(b) If N is a normal subgroup of group G, then G/N is also a group under opration of multiplication of cosets. Prove this. 3

(c) Let (z. +) be the additive abelian group of integers. Let $H=\{0, \pm 2, \pm 4, \pm 6, ----\}$ be a subgroup of group Z. Then show that H is a normal subgroup of Z. Obtain the quotient group Z/H and find it's order. 4

3. (d) Prove that if H is normal subgroup of group G, then prove that $aH = Ha ; \forall a \in G$. 3

(e) Let N and M are two normal subgroup of group G, then prove that NM is also normal subgroup of G. 3

(f) Let G be a group in which for some integer $n>1$, $(ab)^n = a^n b^n$ for all a, b $\in$ G, then show that $G^{(n)} = \{x^n / x \in G\}$ is a normal subgroup of group G. 4

UNIT II

4. (a) If N is a normal subgroup of group G; then show that every quotient group G/N is a homomorphic image of the group G. Further show that Kernel of homomorphism is N. 4

(b) If $\phi : G \to G'$ is a honomorphism with Kernel K, then prove that $G' \cong G/K$. 4

(c) Prove that mapping $\phi : G \to G$ defined by $\phi(x) = x^2$ is homomorphism and find its Kernel where G is a group of non zero real numbers w.r.t. multiplication. 2

5. (d) If $\phi : G \to G'$ be homomorphism, then prove that :

(i) $\phi(e) = e'$ and

(ii) $\phi(x^{-1}) = (\phi(x))^{-1} ; \forall x \in G$. 3

(e) Let $G \to G'$ be a onto homomorphism with Kernel K. If N' is normal subgroup of G' and $N = \{x \in G / \phi(x) \in N'\}$, then prove that :

$$G/N \cong G'/N' \text{ i.e. } G/N \cong \frac{G/K}{N/K}.$$ 5