AT–375

B.Sc. (Part-III) Semester-V Examination

MATHEMATICS (NEW)

(Mathematical Methods)

Paper—X

Time : Three Hours] [Maximum Marks : 60

Note :— Question No. **1** is compulsory and attempt it once and solve **ONE** question from each unit.

1. Choose correct alternative (**1** mark each) : 10

(1) The value of $P_n'(1)$ is :

(a) n (b) n + 1

(c) n(n + 1) (d) $\frac{1}{2}n(n+1)$

(2) If $P_n(x) = x$, then the value of n is :

(a) 0 (b) –1

(c) 1 (d) None

(3) The value of $[J_{1/2}(x)]^2 + [J_{-1/2}(x)]^2$ is :

(a) $\frac{2}{\pi x}$ (b) $\frac{\pi x}{2}$

(c) $\frac{\pi}{2}$ (d) Zero

(4) Each eigen function $y_n(x)$ corresponding to the eigen values λ_n (n = 1, 2, ...) has exactly ______ zeros in (a, b).

(a) n – 1 (b) n

(c) n + 1 (d) One

(5) The function cos x has period :

(a) 2π
(b) π
(c) $\frac{\pi}{2}$
(d) None

(6) If the Fourier series correspond to an odd function f(x) in [–L, L], then its expansion contains only :

(a) constant terms
(b) cosine terms
(c) sine terms
(d) All above

(7) If $L[f(t)] = \frac{3}{s^2 + 9}$ for s > 0, then f(t) is :

(a) sin t
(b) sin 2t
(c) sin 3t
(d) None

(8) The inverse Laplace transform of $\frac{1}{s^2 + a^2}$ is :

(a) sin at
(b) $\frac{1}{a}$ sin at
(c) $\frac{1}{a}$ sin t
(d) sin t

(9) Shifting property of the Fourier transform is :

(a) $F[f(x - a)] = F(\lambda)$
(b) $F[f(x - a)] = F\left(\frac{\lambda}{a}\right)$
(c) $F[f(x - a)] = e^{\lambda a} F(a)$
(d) $F[f(x - a)] = e^{-i\lambda a} \cdot F(\lambda)$

(10) The Fourier transform of f(x) . cos ax is :

(a) $F(\lambda + a) + F(\lambda - a)$
(b) $\frac{1}{2}[F(\lambda + a) + F(\lambda - a)]$
(c) $\frac{1}{2}[F(\lambda + a) - F(\lambda - a)]$
(d) None

UNIT—I

2. (a) Prove that :

$$\int_{-1}^{1} [P_n(x)]^2 \, dx = \frac{2}{2n+1} \text{ if } m = n.$$ 5

(b) Use Rodrigues formula to find $P_n(x)$, n = 0, 1, 2, 3, 4. 5

3. (p) Prove that :

$(2n + 1)\, xP_n = (n + 1)P_{n+1} + nP_{n-1}$. 5

(q) Prove that :

$$\int_{-1}^{1} (x^2 - 1)\, P_n' \; P_{n+1} \, dx = \frac{2n(n+1)}{(2n+1)(2n+3)}$$ 5

UNIT—II

4. (a) Prove that :

$$\frac{d}{dx}[J_p(x)] = \frac{1}{2}[J_{p-1}(x) - J_{p+1}(x)].$$ 5

(b) Prove that :

$xJ_p' = -p\, J_p + x\, J_{p-1}$. 5

5. (p) Express $J_5(x)$ in terms of $J_0(x)$ and $J_1(x)$. 5

(q) Prove that the eigen values of SL problem are real. 5

UNIT—III

6. (a) Obtain the Fourier series for $f(x) = |x|$ in $(-\pi, \pi)$. Hence show that :

$$\frac{\pi^2}{8} = \frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \ldots\ldots\ldots.$$ 5

(b) Expand $f(x) = 2x - x^2$ in the range (0, 3) as a Fourier series with period 3. 5

7. (p) Obtain the Fourier cosine series for $f(x) = x^2$ in $0 < x < 2$. 5

(q) Obtain the Fourier series for $f(x) = \cos\frac{x}{2}$ in $-\pi \le x \le \pi$. 5

UNIT—IV

8. (a) Find :

$$L[\cosh^4 t].$$ 3

(b) Use the transformations of derivatives to find the Laplace transform of cos at. 3

(c) If $L[f(t)] = F(s)$, then prove that $L\left[\frac{f(t)}{t}\right] = \int_{s}^{\infty} F(s)\,ds$, provided the integral exists. 4

9. (p) Find the inverse Laplace transform of $\frac{s}{s^4 - a^4}$. 3

(q) Find the inverse Laplace transform of $\frac{1}{(s-2)(s+2)^2}$ by convolution theorem. 3

(r) Use Laplace transform to solve the equation :

$$\frac{d^2y}{dt^2} + 2\frac{dy}{dt} + y = 3te^{-t}, \quad y(0) = 4, \; y'(0) = 2.$$ 4

UNIT—V

10. (a) Find the finite Fourier sine and cosine transforms of $f(x) = x^2$, $0 < x < l$. 5

(b) Find the Fourier sine and cosine transforms of x^{n-1}, $n > 0$. 5

11. (p) Find Fourier sine transform of $f(x) = e^{-|x|}$, $x \geq 0$. Hence show that :

$$\int_{0}^{\infty} \frac{x \sin mx}{1+x^2}\,dx = \frac{\pi}{2} e^{-m}, \quad m > 0.$$ 5

(q) Show that finite Fourier sine and cosine transforms and their inverses are all Linear transformations. 5