AU–130

B.Sc. Part—III (Semester—V) Examination
5S : MATHEMATICS (New)
(Mathematical Methods)
Paper—X

Time : Three Hours] [Maximum Marks : 60

Note :— Question No. **1** is compulsory and attempt it **once** and solve **one** question from each unit.

1. Choose the correct alternative (**1** mark each) :

(i) If $P_n(x) = (-1)^n$, then what is the value of x ?

(a) 1 (b) –1

(c) 0 (d) None

(ii) All roots of $P_n(x) = 0$ are :

(a) Distinct (b) Equal

(c) Complex (d) None

(iii) What is the value of $J_{-1/2}\left(\frac{\pi}{2}\right)$?

(a) –1 (b) 1

(c) 0 (d) π

(iv) Eigen functions corresponding to different eigen values are :

(a) Linearly dependent (b) Linearly independent

(c) Real (d) None

(v) The fundamental period of tan x is :

(a) π (b) 2π

(c) $\frac{\pi}{2}$ (d) None

(vi) Fourier series are associated with :

(a) Algebraic functions

(b) Special functions

(c) Periodic functions defined on some interval I

(d) Linear functions

(vii) The inverse Laplace transform of $\frac{1}{s-a}$ is :

(a) 1 (b) t

(c) e^t (d) e^{at}

(viii) The Laplace transform of cost is :

(a) $\frac{1}{s^2+1}$ (b) $\frac{s}{s^2+1}$

(c) $\frac{1}{s^2-1}$ (d) $\frac{s}{s^2-1}$

(ix) If $F[f(x)] = F(\lambda)$, then the Fourier transform of f(ax) is :

(a) $F\left(\frac{\lambda}{a}\right)$ (b) $\frac{1}{|a|} F\left(\frac{\lambda}{a}\right), a = 0$

(c) $\frac{1}{|a|} F(\lambda), a \neq 0$ (d) $\frac{1}{|a|} F\left(\frac{\lambda}{a}\right), a \neq 0$

(x) The Fourier transform of convolution of f(x) and g(x) for $-\infty < x < \infty$ is :

(a) $F[f * g] = F[f(x)] \cdot F[g(x)]$ (b) $F[f * g] = F[f(x)] + F[g(x)]$

(c) $F[f * g] = F[f(x)] - F[g(x)]$ (d) $F[f * g] = F[f(x)] / F[g(x)]$ 10

UNIT—I

2. (a) Find $P_3(x)$ by Rodrigues formula and show that $\int_{-1}^{1} x^3 P_3(x)\,dx = \frac{4}{35}$. 2+3

(b) Show that $\int_{-1}^{1} [P_n'(x)]^2\,dx = n(n+1)$. 5

3. (p) Prove that $nP_n = xP_n' - P_{n-1}'$, where $P_n' = \frac{dP_n}{dx}$. 5

(q) Show that $\int_{-1}^{1} x^m P_n(x)\,dx = 0$ if $m < n$. 5

UNIT—II

4. (a) Prove that $xJ'_p = -pJ_p + xJ_{p-1}$. 5

(b) Prove that $\frac{d}{dx}\left[x^p J_p(x)\right] = x^p J_{p-1}(x)$. 5

5. (p) Prove that $J_p(-x) = (-1)^p J_p(x)$, if p is an integer. 5

(q) Prove that the eigen values of the SL problem are real. 5

UNIT—III

6. (a) Obtain the Fourier series for $f(x) = x \cos x$ in $[-\pi, \pi]$. 5

(b) Obtain the Fourier sine series for $f(x) = x^2$ in $0 < x < 2$. 5

7. (p) Express the function $f(x) = \pi x - x^2$ as Fourier sine series in $0 \le x \le \pi$. Deduce that :

$$\frac{1}{1^3} - \frac{1}{3^3} + \frac{1}{5^3} - \frac{1}{7^3} + \ldots = \frac{\pi^3}{32}.$$ 5

(q) Express $f(x) = x$ as a half range sine series in $0 < x < 2$. 5

UNIT—IV

8. (a) Find $L[t^n]$, where n is a positive integer. 3

(b) Using transformations of derivatives find $L[t \sin at]$. 3

(c) Find the Laplace transform of $\int_0^t \frac{e^t \sin t}{t} dt$. 4

9. (p) Find $L^{-1}\left[\frac{s^2 - 3s + 4}{s^3}\right]$. 3

(q) Find the inverse Laplace transform of $\frac{1}{s(s^2+4)}$ by the convolution theorem. 3

(r) Use Laplace transform to find the solution of the equation :

$$\frac{d^2x}{dt^2} + 9x = \cos 2t,\ x(0) = 1,\ x\left(\frac{\pi}{2}\right) = -1.$$ 4

UNIT—V

10. (a) Find the finite Fourier sine and cosine transforms of $f(x) = e^{ax}$ in $(0, \ell)$. 5

(b) Find $F\left[e^{-|x|}\right]$ and hence show that :

$$F\left[e^{-2|x|}\right] = \frac{4}{4 + \lambda^2}.$$ 5

11. (p) Find the Fourier sine transform of $f(x) = \frac{e^{-ax}}{x}$, $a > 0$. Hence evaluate $\int_0^\infty \tan^{-1}\frac{x}{a} \cdot \sin x \, dx$. 5

(q) Find the Fourier sine and cosine transforms of $f(x) = x^n e^{-ax}$, $n > 0$. 5

www.sgbauonline.com