AT–373

B.Sc. (Part-III) Semester-V Examination

MATHEMATICS (NEW)

Mathematical Analysis

Paper—IX

Time : Three Hours] [Maximum Marks : 60

Note :— (1) Question No. **1** is compulsory and attempt it once only.

(2) Attempt **ONE** question from each unit.

1. Choose the correct alternatives :

(i) Consider P = (1, 2, 4) be a partition of interval [1, 4] then $\mu(P)$ is : 1

(a) 1 (b) 0

(c) 2 (d) 4

(ii) Let f be a bounded function defined on [a, b] and p be any partition of [a, b] then L(p, –f) is : 1

(a) –U(p, f) (b) –L(p, f)

(c) L(p, f) (d) U(p, f)

(iii) An integral $\int_0^{\infty} e^{-rx}\,dx$ is convergent if : 1

(a) r < 0 (b) r > 0

(c) r = 0 (d) None of these

(iv) The value of $\Gamma(1/2)$ is : 1

(a) 1/2 (b) 1

(c) $\sqrt{\pi}$ (d) π

(v) If f(z) = (x + ay) + i(bx + y) is analytic then : 1

(a) a = b (b) a + b = 0

(c) a = 1, b = 0 (d) a > b

(vi) Let $f(z) = u + iv$ be analytic function and $z = re^{i\theta}$ then C-R equations are : 1

(a) $u_r = v_\theta,\ u_\theta = -v_r$

(b) $u_r = rv_\theta,\ u_\theta = -\frac{1}{r} v_r$

(c) $u_r = \frac{1}{r} v_\theta,\ v_r = -\frac{1}{r} u_\theta$

(d) $u_r = v_\theta,\ u_\theta = v_r$

(vii) A Mobius transformation which is not identity can have the following number of fixed points : 1

(a) 5

(b) 4

(c) 3

(d) 2

(viii) A bilinear transformation with two non-infinite fixed points p and q have normal form $\frac{w-p}{w-q} = k\left(\frac{z-p}{z-q}\right)$ then BT is elliptic transformation if : 1

(a) $|k| = 1$

(b) $|k| \neq 1$

(c) $|k| = 0$

(d) $|k| = 2$

(ix) For any finite collection $A_1, A_2, \ldots, A_n$ of open sets $\bigcap_{\alpha=1}^{n} A_\alpha$ is : 1

(a) Closed

(b) Open

(c) Semi open

(d) None of these

(x) Every neighbourhood of a point is : 1

(a) Closed

(b) Finite

(c) Open

(d) ϕ

UNIT—I

2. (a) Let a bounded function f defined on [a, b] is integrable on [a, b] iff for each $\epsilon > 0$ there exist a partition P of [a, b] such that $U(p, f) - L(p, f) < \epsilon$. Prove this. 5

(b) Let the function f(x) be defined as $f(x) = \begin{cases} 1, & x \text{ is rational} \\ -1, & x \text{ is irrational.} \end{cases}$ Show that f is not R-integrable over [0, 1], but ' f ' $\in$ R [0, 1]. 5

3. (a) If $f \in R[a, b]$, then prove that $F : [a, b] \to R$ defined by $F(x) = \int_a^x f(t)\, dt$ is continuous on $[a, b]$. If f is continuous at $x_o \in [a, b]$, then prove that F is differentiable at x_o with $F'(x_o) = f(x_o)$. 5

(b) Prove that every continuous function is integrable. 5

UNIT—II

4. (a) Let $f(x), g(x) \in C$, $a \le x < \infty$ and $0 \le f(x) \le g(x)$, $\forall\ x \ge a$. Then prove that :

(i) $\int_a^\infty g(x)\, dx < \infty \Rightarrow \int_a^\infty f(x)\, dx < \infty$ and

(ii) $\int_a^\infty f(x)\, dx = \infty \Rightarrow \int_a^\infty g(x)\, dx = \infty$. 4

(b) Show that $\int_2^\infty \frac{x^2}{\sqrt{x^7+1}}\, dx$ is convergent. 3

(c) Show that $\int_1^\infty \frac{\sin x}{x^2}\, dx$ converges absolutely. 3

5. (a) Prove that :

$$\Gamma 1/2 = \sqrt{\pi}.$$ 4

(b) Evaluate :

$$\int_0^\infty \frac{x^8(1-x^6)}{(1+x)^{24}}\, dx.$$ 3

(c) Show that :

$$\beta(m, n) = 2\int_0^{\pi/2} \sin^{2m-1}\theta \cos^{2n-1}\theta\, d\theta.$$ 3

UNIT—III

6. (a) Prove that a necessary condition that $f(z) = u + iv$ be analytic in a region D is that $u_x = v_y$ and $u_y = -v_x$. 5

(b) Show that the function $w = e^z$ is analytic function and find $\frac{dw}{dz}$. 5

7. (a) If the function $f(z) = u + iv$ is analytic in D then prove that families of curves $u(x, y) = c_1$ and $v(x, y) = c_2$ form an orthogonal system, where c_1 and c_2 are constants. 4

(b) If $f(z)$ and $f'(z)$ are analytic functions then prove that $f(z)$ is constant. 3

(c) Show that $w = e^{\bar{z}}$ is not analytic function for any z. 3

UNIT—IV

8. (a) Prove that the bilinear transformation is a combination of translation, rotation, stretching an inversion transformation. 5

(b) Consider the transformation $w = ze^{i\pi/4}$ and determine the region in the w-plane corresponding to the triangular region bounded by the lines $x = 0$, $y = 0$ and $x + y = 1$ in the z-plane. 5

9. (a) Prove that every bilinear transformation with single non-infinite fixed point α can be put in the normal form $\frac{1}{w-\alpha} = \frac{1}{z-\alpha} + k$, where k is constant. 5

(b) Find the bilinear transformation which maps the points $z = 1, i, -1$ into the points $w = 0, 1, \infty$. 5

UNIT—V

10. (a) Let the mapping $d : c[0, 1] \times c[0, 1] \to R$ be defined by $d(f, g) = \int_0^1 |f(x) - g(x)|\,dx$.

Show that d is metric on $c[0, 1]$. 5

(b) Let X be a metric space. If $\{x_n\}$ and $\{y_n\}$ are sequences in X such that $x_n \to x$ and $y_n \to y$, then show that $d(x_n, y_n) \to d(x, y)$. 5

11. (a) Define neighbourhood of a point in a metric space X and prove that every neighbourhood of a point is open set. 5

(b) Prove that every convergent sequence is Cauchy sequence and give an example of sequence which is Cauchy sequence but not convergent. 5