AU–128

B.Sc. Part—III Semester—V Examination
MATHEMATICS (NEW)
(Mathematical Analysis)
Paper—IX

Time : Three Hours] [Maximum Marks : 60

N.B. :— (1) Question No. **1** is compulsory.

(2) Attempt **ONE** question from each Unit.

1. Choose the correct alternatives :—

(i) If $P_1 = (1, 2, 4)$ and $P_2 = (1, 3, 4)$ be two partitions of [1, 4] then common refinement of P_1 and P_2 is : 1

(a) (1, 2, 4) (b) (1, 3, 4)

(c) (1, 4) (d) (1, 2, 3, 4)

(ii) Let F be bounded function defined on [a, b] and P be any partition of [a, b], if $\alpha < 0$ is any real number then $U(P, \alpha f)$ is : 1

(a) $\alpha L(P, f)$ (b) $\alpha U(P, f)$

(c) $U(P, f)$ (d) None of these

(iii) An improper integral $\int_{a+}^{b} \frac{1}{(x-a)^p} dx$ is divergent if : 1

(a) $p \geq 1$ (b) $p < 1$

(c) $p = \frac{1}{2}$ (d) None of these

(iv) $\beta(m, n)$ is : 1

(a) $\frac{\Gamma(m+n)}{\Gamma m \, \Gamma n}$ (b) $\frac{\Gamma m \, \Gamma n}{\Gamma(m+n)}$

(c) $\frac{\Gamma m \, \Gamma n}{\Gamma(m-n)}$ (d) $\Gamma m \, \Gamma n$

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(v) A function u(x, y) is harmonic in D if : 1

(a) $u_{xx} + u_{yy} = 0$ (b) $u_{xx} - u_{yy} = 0$

(c) $u_{xy} + u_{yx} = 0$ (d) $u_{xx} - u_{xy} = 0$

(vi) Let u, v be real valued function defined on $\mathbb{R}^2$ and $f(z) = u + iv$; $\bar{f}(z) = u - iv$. If f is an analytic function and f is not constant, then : 1

(a) $\bar{f}$ is always analytic (b) $\bar{f}$ may or may not be analytic

(c) $\bar{f}$ is never analytic (d) $f + \bar{f}$ is analytic

(vii) A bilinear transformation $w = \frac{az+b}{cz+d}$, is conformal if : 1

(a) $ad - bc = 0$ (b) $a \neq 0,\ b \neq 0$

(c) $ad - bc \neq 0$ (d) $c \neq 0,\ d \neq 0$

(viii) A bilinear transformation with two non-infinite fixed points α & β having Normal form

$\frac{w-\alpha}{w-\beta} = K\left(\frac{z-\alpha}{z-\beta}\right)$ is Hyperbolic if : 1

(a) $|K| = 1$ (b) $|K| \neq 1$, K is real

(c) $|K| \neq 1$, K is not real (d) None of these

(ix) Let (X, d) be metric space and $A \subset X$. A is nonempty the diameter of A is d(A) if A is unbounded then : 1

(a) $d(A) < \infty$ (b) $d(A) = -\infty$

(c) $d(A) = \infty$ (d) $d(A) = 1$

(x) Let A be a nonempty closed subset of metric space (X, d) then A^C is : 1

(a) open (b) closed

(c) ϕ (d) None of these.

UNIT—I

2. (a) Prove that if f(x) is monotonic function in [a, b] then it is integrable on [a, b]. 4

(b) If $f, g \in R[a, b]$ and $f(x) \leq g(x)$, $\forall x \in [a, b]$, then prove that $\int_a^b f(x)dx \leq \int_a^b g(x)dx$. 3

(c) Show that any constant function defined on [a, b] is integrable on [a, b]. 3

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3. (a) If a function F(x) is continuous on [a, b] and F(x) is continuous and differentiable on [a, b] with $F'(x) = F(x)$, $x \in [a, b]$, then prove that $\int_a^b F(x)dx = F(b) - F(a)$. 4

(b) Let f(x) be a bounded function defined on [a, b] with bounds m and M. Then prove that : $m(b-a) \le L(P, f) \le U(P, f) \le M(b-a)$ for any partition P of [a, b]. 3

(c) Define Darboux Upper and Lowers sums for bounded function f(x) defined on [a, b] and find them for function f(x) with bounds $m_1 = 1$, $m_2 = 2$, $m_3 = 3$, $m_4 = 4$ and $M_1 = 2$, $M_2 = 3$, $M_3 = 4$, $M_4 = 5$ for the partition $P = \{1, 3, 4, 5, 6\}$ of [1, 6]. 3

UNIT—II

4. (a) Prove that $\int_a^\infty \frac{1}{x^p} dx$ converges if $p > 1$ and diverges if $p \le 1$ and $a > 0$. 4

(b) Show that $\int_2^\infty \frac{x^3}{\sqrt{x^7 + 1}} dx$ is divergent. 3

(c) Show that $\int_2^\infty \frac{\cos x}{\sqrt{1 + x^3}} dx$ is Absolutely convergent. 3

5. (a) Prove that $\beta(m, n) = \frac{\Gamma m \Gamma n}{\Gamma m+n}$. 4

(b) Show that $\int_0^1 \sqrt{x(1-x)}\, dx = \pi/8$. 3

(c) Prove that $\Gamma n = \int_0^1 \left(\log\frac{1}{x}\right)^{n-1} dx$. 3

UNIT—III

6. (a) If $f(z) = u(r, \theta) + iv(r, \theta)$ is analytic function in D, then prove that $u_r = \frac{1}{r} v_\theta$ and $v_r = -\frac{1}{r} u_\theta$, CR equations in polar coordinates. 5

(b) Using Milne-Thomson method construct analytic function f(z), whose real part is $e^{-x}(x \cos y + y \sin y)$. 5

7. (a) Let $f(z) = u + iv$ be analytic in the region D, where u and v have continuous partial derivatives upto the second order. Then prove that u and v are harmonic functions. 5

(b) If $w = u + iv$ is analytic function in the region R, then prove that $\frac{\partial(u,v)}{\partial(x,y)} = |f'(z)|^2$. 3

(c) If $w = u + iv$ is analytic function in D, then prove that $\frac{dw}{dz} = \frac{\partial w}{\partial x}$. 2

UNIT—IV

8. (a) Prove that the cross-ratio remains invariant under bilinear transformation. 5

(b) Find the image of the rectangle bounded by $x = 0$, $y = 0$, $x = 2$ and $y = 3$ under the transformation $w = e^{i\pi/4} \times \sqrt{2}$. 5

9. (a) Prove that every bilinear transformation with single non-infinite fixed point α can be put in the normal form $\frac{1}{w-\alpha} = \frac{1}{z-\alpha} + K$, where K is a constant. 5

(b) Find the bilinear transformation which maps the points $z = 1, i, -1$ into points $w = i, o, -i$. 5

UNIT—V

10. (a) Let X be an arbitrary non-empty set. Define d by $d(x, y) = \begin{cases} 0 & \text{if } x = y \\ 1 & \text{if } x \neq y \end{cases}$ show that 'd' is metric on X. 5

(b) Let (X, d) be a metric space and $x, y, x', y' \in X$. Show that

$$|d(x, y) - d(x', y')| \leq d(x, x') + d(y, y').$$ 3

(c) Define :—

(i) Limit point

(ii) Interior point of a set A. 2

11. (a) Let Y be a subspace of a complete metric space X. Then prove that Y is complete $\Leftrightarrow$ Y is closed. 5

(b) Prove that every neighborhood of a point is open set. 3

(c) Define :—

(i) Cauchy sequence

(ii) Complete metric space. 2