B.Sc. (Part–III) Semester—V Examination
5S : MATHEMATICS
Paper—IX
(Analysis)

Time : Three Hours] [Maximum Marks : 60

N.B. :— (1) Question No. 1 is compulsory.

(2) Attempt **ONE** question from each unit.

1. Choose the correct alternatives :

(i) An improper integral $\int_{a^+}^{b} \frac{dx}{(x-a)^p}$ converges if :

(a) $p < 0$ (b) $p < 1$

(c) $p > 0$ (d) $p > 1$ 1

(ii) If P = (1, 3, 4, 5, 6) be the partition of [1, 6] with g.l.b. $m_1 = 1$, $m_2 = 2$, $m_3 = 3$ and $m_4 = 4$ then the value of L(P, f) is :

(a) 10 (b) 11

(c) 12 (d) 13 1

(iii) If f(z) = u + iv be analytic in a region D then $u(x, y) = c_1$ and $v(x, y) = c_2$ form :

(a) orthogonal families of curves

(b) orthogonal families of circles

(c) orthogonal families of hyperbolas

(d) None of the above. 1

(iv) Let X be a metric space and $A \subset X$. Then A is closed iff :

(a) each point of A is its limit point

(b) each limit point of A is in A

(c) each point of X is a limit point of A

(d) each point of A is its interior point. 1

(v) Let X be a metric space. Then :

(a) X is closed but not open

(b) X is open but not closed

(c) X is neither open nor closed

(d) X is both open and closed 1

(vi) The boundary of the set

$A = \{(x, y) \in R^2 / 1 < x^2 + y^2 \leq 4\}$

is :

(a) $\{(x, y) \in R^2/\}x^2 + y^2 = 4\}$

(b) $\{(x, y) \in R^2/x^2 + y^2 = 1\}$

(c) $\{(x, y) \in R^2/x^2 + y^2 = 1 \text{ or } x^2 + y^2 = 4\}$

(d) None of the above. 1

(vii) The invariant points of a transformation $w = f(z)$ are given by :

(a) $f(z) = 0$

(b) $f'(z) = 0$

(c) $f(z) = z$

(d) $f(z) = k$ where k is constant 1

11. (p) Prove that closed subsets of compact sets are compact. 5

(q) If A be an infinite subset of a compact set K, then prove that A has a limit point in K. 5

(q) Let D be a region in the z-plane bounded by the lines x = 0, y = 0, x = 2, y = 1. Find its image under the transformation $w = \sqrt{2}\, e^{i\frac{\pi}{4}} \cdot z$. 5

UNIT—IV

8. (a) Prove that every convergent sequence in a metric space is a Cauchy sequence. Is the converse true ? Explain with an example. 5

(b) Let (X, d) be a metric space and $x, y, x', y' \in X$. Show that :

$|d(x, y) - d(x', y')| \le d(x, x') + d(y, y')$. 5

9. (p) Let X be a complete metric space and $\{F_n\}$ be a decreasing sequence of nonempty closed subsets of X such that $\lim_{n\to\infty} d(F_n) = 0$. Then prove that $\bigcap_{n=1}^{\infty} F_n$ contains exactly one point. 6

(q) Let (X, d) be a metric space and $A \subset X$. Then prove that int A is the largest open subset of A. 4

UNIT—V

10. (a) If f is continuous mapping of a compact metric space into a metric space Y then prove that f(x) is compact. 5

(b) Let f be a continuous one-one mapping of a compact metric space X onto a metric space Y. Then prove that the inverse mapping $f^{-1} : Y \to X$ such that $f^{-1}(f(x)) = x$ for every $x \in X$, is continuous and onto. 5

(viii) A bilinear transformation with two non-infinite fixed points $\alpha + \beta$ can be put in the normal form $\frac{w-\alpha}{w-\beta} = K\left(\frac{z-\alpha}{z-\beta}\right)$. Then the bilinear transformation is elliptic if :

(a) $|K| = 1$ (b) $K = 0$

(c) $K = 1$ (d) $K > 1$ 1

(ix) Equation of a circle with centre at z = a and radius r is :

(a) $z - a = r$ (b) $|z - a| = r$

(c) $|z - a| < r$ (d) $|z - a| > r$ 1

(x) Let X be a metric space and $A \subset X$. Then A is closed iff :

(a) A is open (b) A^C is not closed

(c) A^C is open (d) A^C is closed 1

UNIT—I

2. (a) If f is continuous on [a, b] and F is continuous and differentiable on [a, b] with $F'(x) = f(x)$, $x \in [a, b]$ then prove that $\int_a^b f(x)\,dx = F(b) - F(a)$. 4

(b) Let $f(x) = 1$ if x is rational

$= -1$ if x is irrational

Show that f is not R-integrable over [0, 1] but $|f| \in R[0, 1]$. 3

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(c) If f is bounded and integrable over [a, b] and M, m are bounds of f over [a, b] then prove that :

$$m(b-a) \le \int_a^b f(x)\,dx \le M(b-a).$$ 3

3. (p) Show that :

(i) $\int_a^\infty e^{-rx}dx$ coverges if $r > 0$ and diverges if $r \le 0$. 3

(ii) $\int_a^\infty \frac{dx}{x^p}$ converges if $p > 1$ and diverges if $p \le 1$ and $a > 0$. 3

(q) Show that the integrals :

(i) $\int_0^\infty \frac{\cos x\,dx}{\sqrt{1+x^3}}$ and

(ii) $\int_1^\infty \frac{x\,dx}{3x^4+5x^2+1}$ converge absolutely. 4

UNIT—II

4. (a) If $w = f(z) = u(x, y) + iv(x, y)$ be analytic in a region D of z-plane then prove that :

$u_x = v_y$ and $u_y = -v_x$ in D. 5

(b) Prove that $u = x^3 - 3xy^2$ is harmonic and find its harmonic conjugate v and the corresponding analytic function f(z). 5

5. (p) Prove that if $f(z) = u + iv$ be analytic in a region D then u and v are harmonic in D. 3

(q) If $w = f(z) = u(x, y) + iv(x, y)$ be analytic in D then prove that :

$$\frac{\partial(u,v)}{\partial(x,y)} = \left|\frac{dw}{dz}\right|^2.$$ 3

(r) If f(z) be analytic then show that :

$$\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)|f(z)|^2 = 4\left|f'(z)\right|^2$$ 4

UNIT—III

6. (a) Prove that every bilinear transformation with two non-infinite fixed points α and β can be put in the normal form $\frac{w-\alpha}{w-\beta} = K\left(\frac{z-\alpha}{z-\beta}\right)$, where K is constant. 4

(b) Find the bilinear transformation that maps the points $z = 0, -i, -1$ into $w = i, 1, 0$ respectively. 3

(c) Find the fixed points of bilinear transformation $w = \frac{z-1}{z+1}$. Write its normal form and state its type. 3

7. (p) Show that a bilinear transformation is a combination of translation, rotation, magnification and inversion. 5