AT–372

B.Sc. (Part-III) Semester-V Examination

MATHEMATICS (OLD) (UPTO SUMMER-2018)

(Analysis)

Paper—IX

Time : Three Hours] [Maximum Marks : 60

Note :— (1) Question No. **1** is compulsory.

(2) Attempt **ONE** question from each unit.

1. Choose the correct alternatives :

(1) A function F(x, y) is harmonic in D if : 1

(a) $F_{xx} + F_{yy} = 0$ (b) $F_{xx} - F_{yy} = 0$

(c) $F_{xy} + F_{yx} = 0$ (d) None of these

(2) A Bilinear transformation with only one fixed point is : 1

(a) Loxodromic (b) Elliptic

(c) Hyperbolic (d) Parabolic

(3) If $\{A_\alpha\}$ be a finite or infinite collection of sets A_α then $\left[\bigcup_\alpha A_\alpha\right]^C = \ldots\ldots\ldots\ldots$ 1

(a) $\bigcap_\alpha A_\alpha^c$ (b) $\bigcup_\alpha A_\alpha^c$

(c) $\bigcap_\alpha A_\alpha$ (d) $\bigcup_\alpha A_\alpha$

(4) If f be a bounded function defined on [a, b] and p be any partition of [a, b] then L(p, –f) = ... 1

(a) L(p, f) (b) U(p, f)

(c) –L(p, f) (d) –U(p, f)

(5) A bounded function f is Riemann integrable on [a, b] if its : 1

(a) Upper and lower integrals are equal

(b) Lower and upper integrals are not equal

(c) $L(p, f) = -U(p, f)$

(d) None of these

(6) The fixed points of the transformation $w = \frac{z-1}{z+1}$ are : 1

(a) $z = 1, -1$ (b) $z = i, -i$

(c) $z = 0, 1$ (d) $z = 1, 2$

(7) The neighbourhood of a point is : 1

(a) an open set (b) a closed set

(c) a semi open set (d) a semi closed set

(8) Cauchy Riemann equations of an analytic function $w = u + iv$ are : 1

(a) $u_x = v_y$ and $u_y = v_x$ (b) $u_x = v_x$ and $u_y = v_y$

(c) $u_x = v_y$ and $u_y = -v_x$ (d) $u_x = -v_y$ and $u_y = v_x$

(9) If $f(z)$ and $f(\overline{z})$ are both analytic, then $f(z)$ is : 1

(a) Identically zero (b) Constant

(c) Unbounded (d) None of these

(10) If $f : x \to y$ is a continuous mapping and X is compact, then : 1

(a) $f(x)$ is connected (b) $f(x) = \phi$

(c) $f(x) \neq \phi$ (d) $f(x)$ is compact

UNIT—I

2. (a) If f is bounded function defined on [a, b] and p be any partition of [a, b] then prove that :

(i) $U(p, -f) = -L(p, f)$,

(ii) $L(p, -f) = -U(p, f)$. 4

(b) If f be a bounded and integrable function defined on [a, b] with m, M as infimum, supremum respectively, then prove that there exist a number μ between m and M, such that :

$$\int_a^b f(x)\,dx = \mu(b-a).$$ 3

(c) Prove that $\int_2^\infty \frac{x^2 dx}{\sqrt{x^7+1}}$ converges but $\int_2^\infty \frac{x^3 dx}{\sqrt{x^7+1}} = \infty$. 3

3. (p) Prove that a bounded function f defined on [a, b] is integrable on [a, b] iff for each $\in > 0$, $\exists$ a partition p of [a, b] such that $U(p, f) - L(p, f) < \in$. 4

(q) Test the integrals for convergence :

(i) $\int_0^\infty \frac{x}{x^2+1}\,dx$, 3

(ii) $\int_{-7}^\infty \frac{x^2-1}{x^2+1}\,dx$. 3

UNIT—II

4. (a) If $w = f(z) = u + iv$ be analytic in D and $z = \varepsilon e^{i\theta}$, where u, v, ε, θ are the real numbers then prove that $\frac{\partial u}{\partial \varepsilon} = \frac{1}{\varepsilon}\frac{\partial v}{\partial \theta}$ and $\frac{\partial v}{\partial \varepsilon} = -\frac{1}{\varepsilon}\frac{\partial u}{\partial \theta}$. 5

(b) Separate log z into real and imaginary parts. Using Cauchy-Riemann conditions to show that log z is analytic for $z \neq 0$. 5

5. (p) Show that the function $u = x^3 - 3xy^2$ is harmonic and find the corresponding analytic function. 5

(q) Let $f(z) = \frac{x^2 y^5 (x+iy)}{x^4 + y^{10}}$, $z \neq 0$ and $f(0) = 0$ show that f(z) is not analytic at $z = 0$. 5

UNIT—III

6. (a) Prove that every bilinear transformation with two non-infinite fixed points α, β is of the form $\frac{w-\alpha}{w-\beta} = k\left(\frac{z-\alpha}{z-\beta}\right)$, where K is constant. 5

(b) Find a bilinear transformation which maps point $z = 0, -i, -1$ into $w = i, 1, 0$ respectively. 5

7. (p) Prove that the bilinear transformation is a combination of translation, rotation, stretching and inversion. 5

(q) Determine the equation of the curve in the w-plane into which the straight line $x + y = 1$ is mapped under the transformation (i) $w = z^2$, (ii) $w = 1/z$. 5

UNIT—IV

8. (a) Show that $|d(x,y) - d(x',y')| \leq d(x, x') + d(y, y')$, where $x, y, x', y' \in X$, (X, d) be a metric space. 5

(b) Define neighbourhood of point and show that every neighbourhood is an open set. 5

9. (p) Prove that every convergent sequence in a metric space is a Cauchy sequence. 5

(q) Let (X, d) be a metric space. Prove that a subset G of X is open iff it is a union of open spheres. 5

UNIT—V

10. (a) Prove that closed subsets of compact sets are compact. 5

(b) Prove that a mapping f of a metric space X into a metric space Y is continuous on X if and only if $f^{-1}(V)$ is open in X for every open set V in Y. 5

11. (p) Let X, Y be metric spaces and $f : X \to Y$. Prove that f is continuous iff $f(\overline{A}) \subset \overline{f(A)}$ for every subset A of X. 5

(q) If f be continuous mapping of a connected metric space X into a metric space Y, then prove that f(x) is connected. 5