AR – 579

Fifth Semester B. Sc. (Part – III) Examination

5S : MATHEMATICS

Paper – IX

(Analysis)

P. Pages : 8

Time : Three Hours] [Max. Marks : 60

Note : (1) Question No. **one** is compulsory and attempt once.

(2) Attempt **one** question from each unit.

1. Choose the correct alternative :—

(i) If $P = (1, 3, 4, 5, 6)$ be the partition of $[1, 6]$ with least upper bounds $M_1 = 2$, $M_2 = 3, M_3 = 4, M_4 = 5$ of a bounded function f on $[1, 6]$ then the value of $U(p, f)$ is :

(a) 20

(b) 18

(c) 16

(d) 14. 1

(ii) $\int_1^\infty \frac{dx}{x^3}$ converges to :

(a) 1

(b) $\frac{1}{2}$

(c) -1

(d) $-\frac{1}{2}$. 1

(iii) If $f(z)=u+iv$ be analytic in a region D then $u(x,y)=C_1$ and $v(x,y)=C_2$ form :

(a) Orthogonal families of curves

(b) Orthogonal families of circles

(c) Orthogonal families of hyperbolas

(d) None of the above. 1

(iv) If $w=u+iv$ is analytic in D, then Cauchy–Riemann equations in polar co–ordinates are :

(a) $\frac{\partial u}{\partial r}=-\frac{1}{r}\frac{\partial v}{\partial \theta}$; $\frac{\partial v}{\partial r}\frac{1}{r}=\frac{\partial u}{\partial \theta}$

(b) $\frac{\partial u}{\partial r}=\frac{1}{r}\frac{\partial v}{\partial \theta}$; $\frac{\partial v}{\partial r}=-\frac{1}{r}\frac{\partial u}{\partial \theta}$

(c) $\frac{\partial v}{\partial r}=-r\frac{\partial u}{\partial \theta}$; $\frac{\partial u}{\partial r}=r\frac{\partial v}{\partial \theta}$

(d) $\frac{\partial u}{\partial r}=-r\frac{\partial v}{\partial \theta}$; $\frac{\partial v}{\partial r}=r\frac{\partial u}{\partial \theta}$ 1

(v) Every bilinear Transformation with two non – infinite fixed points α, β is of the form

(a) $\frac{1}{w-\alpha} = k \cdot \frac{1}{w-\beta}$

(b) $\frac{1}{w-\alpha} = -k \frac{1}{w-\beta}$

(c) $\frac{z-\alpha}{w-\alpha} = k \frac{z-\beta}{w-\beta}$

(d) $\frac{w-\alpha}{w-\beta} = k \frac{z-\alpha}{w-\beta}$ 1

(vi) Fixed points of the transformation $w = \frac{z-1}{z+1}$ are :

(a) $\pm i$

(b) $z \pm i$

(c) $1 \pm i$

(d) $1 \pm 2i$. 1

(vii) Every convergent sequence in a metric space is a :

(a) Cauchy sequence

(b) Unbounded sequence

(c) divergent sequence

(d) None of these. 1

(viii) Equation of circle with centre at $z = a$ and radius r is :

(a) $| z + a | = r$

(b) $| z - a | = r$

(c) $| z + a | = - r$

(d) $| z - a | = - r$. 1

(ix) If E is an infinite subset of a compact set K, then :

(a) E has interior point

(b) E has a limit point in K

(c) E has not interior point

(d) E has not a limit point in K. 1

(x) If $A \subset X$, then A is said to be open if :

(a) Every point of A is an interior point of A.

(b) Every point of A is boundary point of A.

(c) Every point of A is not an interior point of A.

(d) None of these. 1

UNIT I

2. (a) If f is continuous on $[a,b]$ and F is continuous and differentiable on $[a,b]$ with $F'(x)=f(x), x\in[a,b]$ then prove that

$$\int_a^b f(x)\,dx = F(b) - F(a).$$ 4

(b) If f be bounded and integrable function defined on $[a,b]$ with m, M as infimum, supremum respectively, then prove that there exists a number μ between m and M such that

$$\int_a^b f(x)\,dx = \mu\,(b-a)$$ 3

(c) Show that any constant function defined on a bounded closed interval is integrable. 3

3. (p) If $f(x)\in C, a \leq x < \infty$ and $\lim_{x\to\infty} x^p f(x) = A$

then prove that $p>1, A\in R \Rightarrow \int_a^\infty f(x)\,dx$ converges absolutely. 4

(q) Test the convergence of

(i) $\int_2^\infty \frac{1}{\sqrt{x^2-1}}\,dx$ (ii) $\int_1^\infty \frac{e^{-x}}{x}\,dx$.

3 + 3

UNIT II

4. (a) Show that the function $f(z) = \sqrt{|xy|}$ is not analytic at the origin, although Cauchy – Riemann equations are satisfied at that point. 5

(b) If $w = f(z) = u + iv$ be analytic in domain D, and $z = re^{i\theta}$, where u, v, r, θ, are the real numbers then prove that

$$\frac{\partial u}{\partial r} = \frac{1}{r}\frac{\partial v}{\partial \theta} \text{ and } \frac{\partial v}{\partial r} = -\frac{1}{r}\frac{\partial u}{\partial \theta}.$$ 5

5. (p) If $u = e^x \cdot \cos y$ then show that $u_{xx} + u_{yy} = 0$. Find the corresponding analytic function. 3

(q) Prove that $w = e^z$ is analytic and $\frac{dw}{dz} = e^z$. 2

(r) Show that $u = 2x - x^3 + 3xy^2$ is harmonic and find its harmonic conjugate function. Hence find the analytic function $f(z) = u + iv$. 5

UNIT III

6. (a) Show that a bilinear transformation is a combination of translation, rotation, magnification and inversion. 5

(b) Find the fixed points of the transformation $w = \dfrac{z-1}{z+1}$. State whether it is hyperbolic, elliptic or loxodromic. 5

7. (p) Find the bilinear transformation which maps the points $z = 1, -1, \infty$ into the points $w = i+1, 1-i, 1$. 5

(q) Prove that every bilinear transformation with a single non – infinite fixed point α can be put in the normal form

$$\frac{1}{w-a} = \frac{1}{z-\alpha} + K \text{, where K is a constant.}$$ 5

UNIT IV

8. (a) Prove that every convergent sequence in a metric space is a Cauchy sequence. Is the converse of above theorem is true ? Give an example to support your answer. 3+3

(b) Prove that Every neighbourhood is an open set. 4

9. (p) Let X be an arbitrary non empty set. Define D by $d(x, y) = \begin{cases} 0, & x = y \\ 1, & x \neq y \end{cases}$

Then show that d is a metric on X. 6

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(q) If $\{x_n\}$ and $\{y_n\}$ are sequences in metric space X such that $x_n \to x$ and $y_n \to y$. Then show that $d(x_n, y_n) \to d(x, y)$. 4

UNIT V

10. (a) Let X, Y be metric spaces and $f : X \to Y$. Then prove that f is continuous iff $\overline{f^{-1}(B)} \subseteq f^{-1}(\bar{B})$ for every subset B of Y. 5

(b) Prove that every totally bounded metric space is bounded. 5

11. (p) Let X, Y be metric spaces and $f : x \to y$. Prove that f is continuous iff $f^{-1}(B') \subseteq [f^{-1}(B)]'$ for every subset B of y, B' = int B. 4

(q) Show that $A = (0, 1)$ is not compact. 3

(r) If B is closed and K is compact then prove that $B \cap K$ is compact. 3