**AS–1448**

**B.Sc. (Part—III) Semester—V Examination**
**MATHEMATICS**
**Paper—IX**
**(Analysis)**

Time : Three Hours] [Maximum Marks : 60

**N.B.** :— (1) Question No. **1** is compulsory.
(2) Attempt **ONE** question from each unit.

1. Choose the correct alternatives :—

(i) $\int_1^\infty \frac{dx}{x^3}$ converges to : 1

(a) $\frac{1}{2}$ (b) 1

(c) 2 (d) 3

(ii) If f be a bounded function defined on [a, b] and p be any partition of [a, b] then U(p, -f) is : 1

(a) L(p, f) (b) U(p, f)

(c) –L(p, f) (d) –U(p, f)

(iii) If f(z) and $f(\bar{z})$ are both analytic, then f(z) is : 1

(a) Unbounded (b) Constant

(c) Identically zero (d) None of these

(iv) A function F(x, y) is harmonic in D if : 1

(a) $F_{xx} + F_{yy} = 0$ (b) $F_{xx} - F_{yy} = 0$

(c) $F_{xy} + F_{yx} = 0$ (d) None of these

(v) If the transformation $w = \frac{2z+3}{z-4}$ transforms the circle $x^2 + y^2 - 4x = 0$ into S, then S is : 1

(a) A circle
(b) A straight line
(c) The region $R_e(w) \geq 0$
(d) The region $R_e(w) \leq 0$

(vi) A Bilinear transformation with only one fixed point is : 1

(a) Loxodromic
(b) Elliptic
(c) Hyperbolic
(d) Parabolic

(vii) If $\{A_\alpha\}$ be a finite or infinite collection of sets $A_\alpha$ then $\left[\bigcup_\alpha A_\alpha\right]^c =$ 1

(a) $\bigcap_\alpha A_\alpha^c$
(b) $\bigcup_\alpha A_\alpha^c$
(c) $\bigcap_\alpha A_\alpha$
(d) $\bigcup_\alpha A_\alpha$

(viii) In the real line R, which of the following is true ? 1

(a) Every Cauchy sequence is convergent
(b) Every sequence is bounded
(c) Every sequence is convergent
(d) None of these

(ix) A metric space (X, d) is complete if : 1

(a) Every convergent sequence in X is a Cauchy sequence
(b) Every Cauchy sequence in X is convergent in X
(c) Every convergent sequence in X is not a Cauchy sequence
(d) None of these

(x) If B is closed and K is compact, then $B \cap K$ is : 1

(a) Bounded
(b) Closed
(c) Convergent
(d) Compact



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## UNIT—I

2. (a) If f be continuous and integrable on [a, b] then prove that $\int_a^b f(x)\,dx = f(c)\,(b-a)$, where c is some point in [a, b]. 4

(b) If m and M are glb and lub of f(x) in [a, b] then show that

$$m(b-a) \le L(p, f) \le U(p, f) \le M(b-a).$$ 3

(c) If f is bounded function defined on [a, b] and p be any partition of [a, b] then prove that :

(i) $U(p, -f) = -L(p, f)$

(ii) $L(p, -f) = -U(p, f)$. 3

3. (p) Show that :

(i) $\int_0^\infty e^{-rx}\,dx$ converges if $r > 0$ and diverges if $r \le 0$. 3

(ii) $\int_a^\infty \frac{dx}{x^p}$ converges if $p > 1$ and diverges if $p \le 1$ and $a > 0$. 3

(q) Using limit test, show that the integrals :

(i) $\int_2^\infty \frac{x}{1-x^2}\,dx = \infty$ and 2

(ii) $\int_1^\infty \frac{x\,dx}{3x^4+5x^2+1}$ coverges absolutely. 2

## UNIT—II

4. (a) If $w = f(z) = u + iv$ be analytic in D and $z = re^{i\theta}$, where u, v, r, θ are the real numbers then prove that $\frac{\partial u}{\partial r} = \frac{1}{r}\frac{\partial v}{\partial \theta}$ and $\frac{\partial v}{\partial r} = -\frac{1}{r}\frac{\partial u}{\partial \theta}$. 5

(b) Separate sin z into real and imaginary parts. Use Cauchy-Riemann conditions to show that : sin z is analytic. Prove that $\frac{d}{dz}(\sin z) = \cos z$. 5

5. (p) Find an analytic function f(z) such that

$$R_e\{f'(z)\} = 3x^2 - 4y - 3y^2$$

and f(1 + i) = 0, using Milne-Thomson method. 5

(q) If f(z) = u + iv be analytic in the region D, where u and v have continuous partial derivatives upto the second order, then prove that u and v both are harmonic functions. 5

## UNIT—III

6. (a) Prove that every bilinear transformation with two non-infinite fixed points α, β is of the form $\frac{w-\alpha}{w-\beta} = K\left(\frac{z-\alpha}{z-\beta}\right)$, where K is a constant. 5

(b) Find the fixed points of the bilinear transformation $w = \frac{(2+i)z-2}{i+z}$, what is its normal form ? Show that the transformation is Loxodromic. 5

7. (p) Find the image of the rectangle bounded by x = 0, y = 0, x = 2 and y = 3 under the transformation $w = \sqrt{2}\, e^{i\pi/4}.z$ 5

(q) Prove that the cross ratio remains invariant under a bilinear transformation. 5

## UNIT—IV

8. (a) If X be a metric space with metric d then show that $d_1$ defined by

$d_1(x, y) = \frac{d(x, y)}{1+d(x, y)}$, is also a metric on x. 5

(b) If $\{x_n\}$ and $\{y_n\}$ are sequences in a metric space X such that $x_n \to x$ and $y_n \to y$. Then show that $d(x_n, y_n) \to d(x, y)$. 5

9. (p) Prove that the set A is open if and only if its complement is closed. 5

(q) Prove that the union of two nowhere dense sets in a metric space is nowhere dense. 5

## UNIT—V

10. (a) Prove that a mapping f of a metric space X into a metric space Y is continuous on X if and only if $f^{-1}(V)$ is open in X for every open set V in Y. 6

(b) Let $f : R \to R$ such that

$$f(x) = \begin{cases} x & , \ x \text{ is irrational} \\ -x & , \ x \text{ is rational.} \end{cases}$$

Show that f is continuous only at $x = 0$. 4

11. (p) Let X, Y be metric spaces and $f : X \to Y$. Prove that f is continuous iff $f^{-1}(B') \subseteq [f^{-1}(B)]'$ for every subset B of Y, $B' = \text{int } B$. 5

(q) If f be a continuous mapping of a connected metric space X into a metric space Y. Then prove that f(x) is connected. 5