B.Sc. Part—III Semester—V Examination
MATHEMATICS (OLD) (UPTO SUMMER-2018)
(Analysis)
Paper—IX

Time : Three Hours] [Maximum Marks : 60

N.B. :— (1) Question No. **1** is compulsory.

(2) Attempt **ONE** question from each Unit.

1. Choose the correct alternatives :—

(i) A bounded function f is R-integrable on [a, b] if its : 1

(a) upper and lower integrals are equal.

(b) upper and lower integrals are not equal.

(c) L(P, f) = U(P, f).

(d) None of the above.

(ii) An improper integral $\int_a^{b^-} \frac{dx}{(b-x)^p}$ converges if : 1

(a) $p > 1$ (b) $p < 1$

(c) $p \geq 1$ (d) $p \leq 1$

(iii) If f(z) = u + iv be analytic in a region D of z-plane then : 1

(a) u is harmonic but v is not

(b) v is harmonic but u is not

(c) both u & v are harmonic.

(d) neither u is harmonic nor v is harmonic.

(iv) A bilinear transformation w = f(z) having only one fixed point is : 1

(a) elliptic (b) loxodromic

(c) hyperbolic (d) parabolic

(v) Let A be a subset of a metric space X. Then A is open if : 1

(a) For every $x \in A$, there is $r > 0$ such that $N_r(x) \subset A$.

(b) each point of A is its interior point.

(c) for every $x \in A$, there is a neighborhood of x that lies entirely in A.

(d) all of these.

(vi) Let $B = \{x/0 < x < 1$ and x is rational number$\}$ be a subset of metric space R. Then : 1

(a) B is closed in R

(b) B is open in R

(c) B is not open in R

(d) None of these.

(vii) The invariant points of the transformation $w = \frac{3z-4}{z-1}$ are : 1

(a) $1 \pm i$ (b) 2, 2

(c) $2 \pm i$ (d) −2, −2

(viii) Set of Real numbers R is : 1

(a) countable (b) uncountable

(c) finite (d) None of these.

(ix) A neighborhood of a point in a metric space is : 1

(a) an open set (b) a closed set

(c) neither open nor closed (d) both open and closed

(x) A transformation $w = f(z) = e^{i\pi/4} \cdot z$ is a : 1

(a) rotation (b) magnification

(c) translation (d) inversion

UNIT—I

2\. (a) Prove that a bounded function f defined on [a, b] is integrable on [a, b] iff for each $\epsilon > 0$ $\exists$ a partition P of [a, b] such that $U(P, f) - L(P, f) < \epsilon$. 5

(b) If f is a function defined by $f(x) = x$ on [0, 2], then show that f is integrable in Riemann sense over [0, 2] and $\int_0^2 f(x)dx = 2$. 5

3. (p) Let $f(x), g(x) \in C$, $a \le x < \infty$ and $0 \le f(x) \le g(x)$ for every $x \ge a$. Then prove that : 4

(i) $\int_a^\infty g(x)dx$ converges $\Rightarrow \int_a^\infty f(x)dx$ converges

(ii) $\int_a^\infty f(x)dx = \infty \Rightarrow \int_a^\infty g(x)dx = \infty$.

(q) Prove that :

(i) $\int_1^\infty \frac{\sin x}{x^2} dx$ converges absolutely. 2

(ii) $\int_2^\infty \frac{x^3 dx}{\sqrt{x^7+1}}$ diverges. 2

(iii) $\int_2^\infty \frac{x^2 dx}{\sqrt{x^7+1}}$ converges. 2

UNIT—II

4. (a) Prove that the function sin z is analytic and find its derivative. 5

(b) Prove that the Cauchy-Riemann equations in polar form can be written as $u_r = \frac{1}{r} v_\theta$ and $v_r = -\frac{1}{r} u_\theta$, where $w = f(z) = u(x, y) + iv(x, y)$ is a function defined in a region D of Z-plane. 5

5. (p) If $w = f(z) = u + iv$ be analytic in a region D then prove that the families of curves $u(x, y)$ C_1 and $v(x, y) = C_2$ form an orthogonal system of curves, where C_1 and C_2 are arbitrary constants. 5

(q) Find the analytic function whose real part is $e^{-x}(x \sin y - y \cos y)$. 5

UNIT—III

6. (a) Prove that every bilinear transformation with a single non-infinite fixed point α can be put in the normal form :

$$\frac{1}{w-\alpha} = \frac{1}{z-\alpha} + K, \text{ where K is constant.}$$ 5

(b) Find the region D' of w-plane into which the region D in Z-plane bounded by x = 0, y = 0, x + y = 1 is transformed by the mapping $w = e^{i\pi/4} \cdot z$. 5

(p) Show that cross ratio remains invariant under a bilinear transformation. 5

(q) Find the fixed points of the bilinear transformation $w = \frac{(2+i)z - 2}{i+z}$. Write its normal form and show that it is loxodromic. 5

UNIT—IV

8. (a) Define :

(i) interior point of a set

(ii) limit point of a set. 2

(b) Prove that every neighborhood is an open set. 3

(c) Let (X, d) be a metric space. Then show that d_1 defined by :

$$d_1(x, y) = \frac{d(x, y)}{1 + d(x, y)}$$

is also a metric on X. 5

9. (p) Let $Y \subset X$. Then prove that a subset A of Y is open relative to Y iff $A = Y \cap G$ for some open subset G of X. 6

(q) If (X, d) is a metric space and $A \subset X$, then prove that A is closed iff A contains its boundary. 4

UNIT—V

10. (a) Let $K \subset Y \subset X$. Then prove that K is compact relative to X iff K is compact relative to Y. 5

(b) Let X, Y be metric spaces and f : X→Y. Prove that f is continuous iff $f(\overline{A}) \subset \overline{f(A)}$ for every subset A of X. 5

11. (p) Prove that a mapping f of a metric space X into a metric space Y is continuous on X iff $f^{-1}(V)$ is open in X for every open set V in Y. 5

(q) Prove that every totally bounded metric space is bounded. 5