AU–129

B.Sc. Part—III Semester—V Examination

MATHEMATICS (OLD) (UPTO SUMMER-2018)

(Modern Algebra)

Paper—X

Time : Three Hours] [Maximum Marks : 60

N.B. :— Question No. **1** is compulsory and answer **ONE** question from each Unit.

1. Choose the correct alternatives (1 mark each) :—

(1) Idempotent element of group is : 1

(a) Any element of group (b) Identity element of group

(c) All elements of group (d) None of these

(2) The identity element in a quotient group G/N is : 1

(a) Identity element in group (b) Identity element in N

(c) N (d) None of these

(3) An element a of group G is of order n then : 1

(a) $a^n = a \cdot a \cdot a$ .....a n times (b) a^n = Identity

(c) a^n = – Identity (d) None of these

(4) A ring Z_6 is : 1

(a) Commutative ring (b) not an integral domain

(c) not a field (d) all above

(5) The Kernel of a homomorphism $f : G \to G'$ is : 1

(a) a subset of G (b) a subset of G'

(c) {e'} (d) None of these

(6) A homomorphism from a Group G into itself is called : 1

(a) Automorphism (b) Endomorphism

(c) Monomorphism (d) Epimorphism

(7) Which of them is not an integral domain ? 1

(a) $(Q, +, \cdot)$ (b) $(R, +, \cdot)$

(c) $(C, +, \cdot)$ (d) $(N, +, \cdot)$

(8) A ring $(R, +, \cdot)$ is called a Boolean ring $\forall a \in R$: 1

(a) $a^2 = e$ (b) $a^2 = a$

(c) $a = a^{-1}$ (d) $a = e$

(9) The polynomial $f(x) = x^2 - 3$ is : 1

(a) Reducible in the field of rational numbers

(b) Reducible in the field of real numbers

(c) Irreducible in the field of real numbers

(d) None of these

(10) An integral domain is : 1

(a) Always a field (b) Never a field

(c) A field when it is finite (d) None of these

UNIT—I

2. (a) Prove that a subgroup H of a group G is a normal sub-group of G iff the product of two right cosets of H in G is again a right coset of H in G. 5

(b) Prove that the intersection of any two normal subgroups of a group is a normal subgroup. 3

(c) Show that every subgroup of an abelian group is normal. 2

3. (d) Define normal subgroup and prove that a subgroup H of a group G is normal iff $x H x^{-1} = H, \forall x \in G$. 5

(e) If N is normal subgroup of an abelian group G, then prove that the quotient group G/N is abelian. 3

(f) Define :–

(i) Right coset (ii) Left coset

with respect to addition. 2

UNIT—II

4. (a) If f is homomorphism from a group G into a group G'. Then show that the Kernel of f is a normal subgroup of G. 5

(b) Prove that the group G is abelian iff the mapping $f : G \to G'$ defined by $f(a) = a^2$, $\forall a \in G$ is homomorphism. 4

(c) Define Isomorphism. 1

5. (d) If G is a group and N is normal sub group of G and $F : G \to G/N$ defined by $f(x) = Nx$, $\forall x \in G$ then prove that F is homomorphism of G onto G/N and ker F = N. 5

(e) G is a group of non-zero real numbers under multiplication, $\phi : G \to G$ is defined by $\phi(x) = x^2$, $\forall x \in G$. Verify ϕ is homomorphism. Find Kernel of ϕ. 5

UNIT—III

6. (a) Define : A ring with zero divisors. Prove that a ring R is without zero divisors iff cancellation laws hold in R. 1+4

(b) Prove that a nonempty subset S of a ring R is a subring of R iff $x-y, x \cdot y \in S$, $\forall x, y \in S$. 5

7. (c) If R is ring such that $a^2 = a$, $\forall a \in R$, then prove that :

(i) $a + a = 0$

(ii) $a + b = 0 \Rightarrow a = b$, $\forall a, b \in R$. 5

(d) Let $M = \left\{ \begin{bmatrix} a & b \\ c & d \end{bmatrix} \middle/ a, b, c, d \in R \right\}$. Show that M is ring with unity with respect to addition and multiplication of matrices. 5

UNIT—IV

8. (a) Prove that every field is an integral domain. 5

(b) If the characteristic of the ring R is 2 and ab = ba for all $a, b \in R$, then show that :

$(a+b)^2 = a^2 + b^2 = (a-b)^2$. 3

(c) Define :

(i) Subfield

(ii) Prime field. 2

9. (d) Prove that the characteristic of an integral domain is either zero or prime. 4

(e) Show that the set of numbers of the form $a + b\sqrt{2}$, with a and b as rational numbers is a field. 5

(f) Define : Division ring. 1

UNIT—V

10. (a) If $f(x)$ and $g(x) \neq 0$ are any two polynomials over a field F, then show that there exist two unique polynomials $t(x)$ and $r(x)$ over F such that :

$$f(x) = t(x) \cdot g(x) + r(x),$$

where $r(x) = 0$ or $\deg r(x) < \deg g(x)$. 5

(b) If $f(x)$ is a polynomial over a field F and $\alpha \in F$, then show that $(x - \alpha)$ divides $f(x)$ iff $f(\alpha) = 0$. 3

(c) Define :—

(i) Monic polynomial

(ii) Associate Polynomials. 2

11. (d) Prove that any non-constant polynomial in $F(x)$ can be written in a unique manner as a product of irreducible polynomials in $F[x]$, where F is a field. 5

(e) Prove that the polynomial $f(x) = x^4 + 2x + 2$ is irreducible over the field of rational numbers. 3

(f) Use the remainder theorem to determine the remainder when $f(x) = 3x^5 - 4x^4 - 2x + 1 \in R[x]$ is divided by $x + 2 \in R(x)$. 2