AV–1783

B.Sc. (Part—III) Semester—V Examination

5S : MATHEMATICS (New)

(Mathematical Methods)

Paper—X

Time : Three Hours] [Maximum Marks : 60

Note :— Question No. **1** is compulsory and attempt it once and solve **ONE** question from each unit.

1. Choose the correct alternative (**1** mark each) :

(i) If $p_n(x) = 1$, then what is the value of n ?

(a) 1 (b) –1

(c) 0 (d) None

(ii) The integral $\int_{-1}^{1} p_n(x) \cdot p_m(x)\, dx \neq 0$ if :

(a) $m < n$ (b) $m > n$

(c) $m \neq n$ (d) $m = n$

(iii) What is the value of $J_{1/2}\left(\frac{\pi}{2}\right)$?

(a) 0 (b) 1

(c) π (d) $\frac{\pi}{2}$

(iv) The eigen values of Strum-Liouville problem are :

(a) Real (b) Complex

(c) Equal (d) None

(v) Every Fourier series is a :

(a) Trigonometric series (b) Power series

(c) Exponential series (d) None

(vi) The fundamental period of sin x is :

(a) π (b) 2π

(c) $\frac{\pi}{2}$ (d) None

(vii) If $L[f(t)] = \frac{1}{s^2}$, $(s > 0)$, then f(t) is :

(a) t^n (b) t^2

(c) 1 (d) t

(viii) Every bounded function is of exponential order :

(a) 1 (b) –1

(c) 0 (d) 2

(ix) If $F[f(x)] = F(\lambda)$, then Fourier transform of $f(x - a)$ is :

(a) $e^{\lambda a}\cdot F(\lambda)$ (b) $e^{i\lambda a}\cdot F(\lambda)$

(c) $e^{-i\lambda a}\cdot F(\lambda)$ (d) None

(x) The Fourier transform of $e^{-|x|}$ is :

(a) $\dfrac{2}{1+\lambda^2}$ (b) $\dfrac{1}{1+\lambda^2}$

(c) $\dfrac{2}{1-\lambda^2}$ (d) $\dfrac{1}{1-\lambda^2}$ 10

UNIT—I

2. (a) Show that $p_n(1) = 1$ and $p_n(-x) = (-1)^n p_n(x)$. Hence or otherwise deduce that $p_n(-1) = (-1)^n$. 5

(b) Prove that $(2n+1)x\,p_n = (n+1)p_{n+1} + n\,p_{n-1}$. 5

3. (p) Prove that :

(i) $\int_{-1}^{1} p_n(x)\,dx = 0,\ n \neq 0$ 4

(ii) $\int_{-1}^{1} p_0(x)\,dx = 2$. 1

(q) Show that $\int_{-1}^{1} p_m(x)\cdot p_n(x)\,dx = 0$ if $m \neq n$. 5

UNIT—II

4. (a) Prove that $xJ'_p = pJ_p - xJ_{p+1}$. 5

(b) Express $J_5(x)$ in terms of $J_0(x)$ and $J_1(x)$. 5

5. (p) Prove that $J_{-p}(x) = (-1)^p J_p(x)$, if p is a positive integer. 5

(q) Find all the eigen values and eigen functions of the SL problem $y'' + \lambda^2 y = 0,\ y'(0) = y'(\ell) = 0$, $0 \le x \le \ell$. 5

UNIT—III

6. (a) Obtain the Fourier series for $f(x) = x^2$ in $[-\pi, \pi]$. Hence deduce that $\dfrac{\pi^2}{6} = \dfrac{1}{1^2} + \dfrac{1}{2^2} + \dfrac{1}{3^2} + \ldots$ 5

(b) Find the Fourier series for the function f(x) defined in $-\pi < x < \pi$ as :

$$f(x) = \begin{cases} -x & , -\pi < x < 0 \\ x & , 0 < x < \pi \end{cases}.$$

Deduce that :

$$\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \ldots = \frac{\pi^2}{8}.$$ 5

7. (p) Obtain the Fourier series for $\sqrt{1-\cos x}$ in $(0, 2\pi)$. Hence deduce that $\frac{1}{2} = \sum_{n=1}^{\infty} \frac{1}{(4n^2-1)}$. 5

(q) Obtain the cosine half range series for the function $f(x) = x$ in $0 < x < 2$. 5

UNIT—IV

8. (a) If $L[f(t)] = F(s)$, then prove that $L[e^{at} f(t)] = F(s-a)$. 3

(b) Find the Laplace transform of $t^2 \sin at$. 3

(c) Evaluate $\int_0^{\infty} \frac{\cos 6t - \cos 4t}{t} dt$. 4

9. (p) Find the inverse Laplace transform of $\frac{6s-4}{s^2-4s+20}$. 3

(q) Verify the convolution theorem for $f_1(t) = t$, $f_2(t) = \cosh t$. 3

(r) Using Laplace transform method, solve the equation :

$$\frac{d^2y}{dt^2} + 2\frac{dy}{dt} + y = 3te^{-t}, \; y(0) = 4, \; y'(0) = 2.$$ 4

UNIT—V

10. (a) Find the finite Fourier sine and cosine transforms of mx, $0 < x < \ell$. 5

(b) Show that Fourier cosine transform of $f(x) = e^{-x^2}$ is $\frac{1}{\sqrt{2}} e^{-\lambda^2/4}$. 5

11. (p) Find the Fourier sine and cosine transforms of x^{n-1}, $n > 0$. 5

(q) Find the finite Fourier sine and cosine transforms of $f(x) = \sin ax$ in $(0, \pi)$. 5