UNIT – V

10. (a) Prove that

$$\int_{-1}^{1} [P_n(x)]^2 dx = \frac{2}{2n+1} \text{ if } m = n$$ 3

(b) Prove that

$$\int_a^b J_0(x) J_1(x) dx = \frac{1}{2} [J_0^2(a) - J_0^2(b)]$$ 3

(c) Prove that eigen values of the Sturm – Liouville problem are real. 4

9. (p) Prove that $P_n(x) = \frac{1}{2^n n!} \frac{d^n}{dx^n} (x^2 - 1)^n$ 4

(q) Show that Bessel's D. E. is of Sturm – Liouville Type. 3

(r) Prove the recurrence formula

$2J_n^1 = J_{n-1} - J_{n+1}$ 3

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Fourth Semester B. Sc. (Part – II) Examination

4S – MATHEMATICS

Paper – VII

(Laplace Transforms and Fourier Series)

P. Pages : 8

Time : Three Hours] [Max. Marks : 60

Note : (1) Question No. **one** is compulsory and solve this question in **one** attempt only.

(2) Solve **One** question from each unit.

1. Choose the correct alternatives :—

(i) If $L[f(x)] = F(s)$ and $g(x) = \begin{cases} f(x-a), & x > a \\ 0, & x < a \end{cases}$

then $L[g(x)]$ is

(a) $e^{-as} F(s)$

(b) $e^{as} F(s)$

(c) $e^{s} F(s)$

(d) $e^{as} F(s)$. 1

(ii) If $L[f(t)] = F(s)$, then $L[f(at)]$ is

(a) $\frac{1}{a} F\left(\frac{s}{a}\right)$

(b) $F\left(\frac{s}{a}\right)$

(c) $\frac{1}{a} F(s)$

(d) None of these. 1

(iii) If $L^{-1}[f(s)] = f(x)$, then $L^{-1}\left[\frac{F(s)}{s}\right]$ is equal to

(a) $\int_s^x F(u)\, du$

(b) $\int_0^x F(u)\, du$

(c) $\int_s^t F(s)\, ds$

(d) $\int_1^2 f(x)\, dx$ 1

(iv) The value of $L\left[\frac{1}{a} \sin h\, ax\right]$ is

(a) $\frac{1}{a^2 - s^2}$

(b) $\frac{1}{s^2 - a^2}$

(c) $\frac{1}{s - a}$

(d) $\frac{1}{s^3}$ 1

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7. (p) Solve $\int_0^x f(x) f(x-u)\, du = 2f(x) + x - 2$ 5

(q) Find the bounded solution of $u_x = 2ut + u$, $u(x, 0) = e^{-3x}$ for $x > 0, t > 0$. 5

UNIT – IV

8. (a) Find the Fourier series for function f (x) defined in $[-\pi, \pi]$ as

$$f(x) = \begin{cases} -\pi & -\pi < x < 0 \\ x & 0 < x < \pi \end{cases}$$

and deduce that $\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \ldots = \frac{\pi^2}{8}$ 5

(b) Find the Fourier sine series for function $f(x) = x^2$ in $[0, 2]$. 5

9. (p) Find the Fourier series for f (x), where

$$f(x) = \begin{cases} 0 & -2 < x < 0 \\ 1 & 0 < x < 2 \end{cases}$$ 5

(q) Find the Fourier series for $f(x) = |x|$ in $[-\pi, \pi]$.

Deduce that $\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \ldots = \frac{\pi^2}{8}$

5

(b) Obtain the inverse Laplace Transform of

$$\frac{S^2-6}{S^3+4S^2+3S}$$ 4

(c) If $L^{-1}[F(s)]=F(x)$, then show that

$$L^{-1}\left\{\frac{1}{a}F\left(\frac{s}{a}\right)\right\}=f(ax)\ ;\ a>0$$ 3

5. (p) Find the inverse Laplace transform of

(i) $\frac{3s-7}{s^2-2s-3}$ (ii) $\frac{s^2}{(s-4)^3}$ 2+3

(q) State and prove convolution theorem. 1 + 4

UNIT – III

6. (a) Solve the D. E. by Laplace transform method

$$\frac{d^2x}{dt^2}-3\frac{dx}{dt}+2x=4t+e^{3t}\text{ , where } x(0)=1$$

and x (0) = – 1. 4

(b) Solve the simultaneous equations by using L. T.

$$\frac{dx}{dt}-y=e^t \text{ and } \frac{dy}{dt}+x=\sin t\text{, where } x=1,$$

y = 0 at t = 0. 6

(v) If $y=y(x,t)$, then $L(y_t)$ = ?

(a) $x\bar{y}(x,s)+y(x,0)$

(b) $s\bar{y}(x,s)-y(x,0)$

(c) $s\bar{y}(x,0)+y(x,s)$

(d) None of these. 1

(vi) If the equation is

$$f(x)=g(x)+\int_0^x k(x-u)f(u)\,dx,$$

then $L[f(x)]$ is

(a) $\frac{G(s)}{1-K(s)}$

(b) $\frac{K(s)}{1-G(s)}$

(c) $\frac{1-K(s)}{G(s)}$

(d) $\frac{G(s)}{K(s)}$. 1

(vii) Half range Fourier Cosine series for f(x) is

(a) $\frac{2}{l}\int_0^l f(x)\,dx$

(b) $\frac{2}{l}\int_0^l f(x)\ \frac{\sin nx}{l}\,dx$

(c) $\frac{2}{l} \int_0^l f(x) \cos \frac{n\pi x}{l} . dx$

(d) $\int_0^l an \cos nx \, dx$ 1

(viii) If f and g are respectively odd and even functions then $f \cdot g$ is

(a) even function

(b) odd function

(c) odd and even function

(d) none of these 1

(ix) The equation

$$P_n(x) = \frac{1}{2^n n!} \frac{d^n}{dx^n} (x^2 - 1)^n \quad \text{is}$$

(a) Legendre's equation

(b) Bessel's equation

(c) Generating function

(d) Rodrigue's equation. 1

(x) If $J_n(x)$ is the solution of Bessel's D. E. the $J_n(-x)$ is

(a) $-J_n(x)$

(b) 0

(c) $(-x) \, J_n(x)$

(d) $(-1)^n \, J_n(x)$. 1

UNIT - I

2. (a) Find L. T. of f(x) if

$$f(x) = \begin{cases} \sin(x - \frac{\pi}{4}) & x > \frac{\pi}{4} \\ 0 & x < \frac{\pi}{4} \end{cases}$$ 3

(b) Evaluate $\int_0^\infty n \, e^{-3x} \sin x \, dx$ 4

(c) Find Laplace Transform of coshan. 3

3. (p) Prove that $L[x^n f(x)] = (-1)^n \frac{d^n}{ds^n} F(s)$ 4

(q) If $L[f(x)] = F(s)$, then prove that $L[e^{ax} f(x)] = F(s-a)$. 2

(r) If $L\left[\frac{\sin x}{x}\right] = \tan^{-1} \frac{1}{s}$, then prove that

$$L\left[\frac{\sin ax}{x}\right] = \cot^{-1}\left(\frac{s}{a}\right)$$ 4

UNIT - II

4. (a) Verify the convolution theorem for $f_1(x) = x$ and $f_2(x) = \cos hx$. 3