AS–1423

B.Sc. Part—II (Semester—IV) Examination

MATHEMATICS (Old)

(Laplace Transform and Fourier Series)

Paper—VII

Time : Three Hours] [Maximum Marks : 60

Note :— (1) Question No. **1** is compulsory and attempt at once.

(2) Solve **one** question from each unit.

1. Choose the correct alternatives : 1

(i) Laplace transform of function f(x) = 1 is :

(a) $\frac{1}{s+1}, s>0$ (b) s, s > 1

(c) $\frac{1}{s-1}, s>1$ (d) $\frac{1}{s}, s>0$

(ii) If $L^{-1}[F(s)] = f(x)$, then $L^{-1}\left[\frac{d^n}{ds^n} F(s)\right]$ is : 1

(a) $x^n f(x)$ (b) $(-1)^n x^n f(x)$

(c) $f^n(x)$ (d) None of these

(iii) The coefficient in a half range cosine series for the function defined on (0, *l*) is given by the formula : 1

(a) $\int_0^\ell f(x)\sin\frac{n\pi x}{\ell}dx$ (b) $\int_0^\ell f(x)\cos\frac{n\pi x}{\ell}dx$

(c) $\frac{1}{\ell}\int_0^\ell f(x)\sin\frac{n\pi x}{\ell}dx$ (d) $\frac{2}{\ell}\int_0^\ell f(x)\cos\frac{n\pi x}{\ell}dx$

(iv) Let $P_n(x)$ is the solution of Legendre's DE. Then $P_1(x)$ is : 1

(a) x (b) $\frac{1}{x}$

(c) x^2 (d) $x + 1$

(v) For the function u(x, t) defined on [a, b], $t \geq 0$ and L(u) = U(x, s) then L[ut] is given by : 1

(a) $\frac{dU}{dx}$ (b) $\frac{d^2U}{dx^2}$

(c) $SL[u] - u(x, 0)$ (d) None of these

(vi) If $L(f(t)) = F(s)$ then $L(e^{-\alpha t} f(t))$ is : 1

(a) $F(s + \alpha)$ (b) $F(s - \alpha)$

(c) $\frac{1}{s} F\left(\frac{s}{\alpha}\right)$ (d) None of these

(vii) L[t sin at] is equal to : 1

(a) $\frac{2as}{(s^2 + a^2)^2}$ (b) $\frac{2as}{(s^2 - a^2)^2}$

(c) $\frac{2as}{s^2 + a^2}$ (d) $\frac{2as}{s^2 - a^2}$

(viii) If $L^{-1}[F(s)] = f(t)$, then $L^{-1}[F(as)]$ is : 1

(a) $\frac{1}{a} f\left(\frac{a}{t}\right)$ (b) $\frac{1}{a} f\left(\frac{t}{a}\right)$

(c) $\frac{1}{a} f^1\left(\frac{a}{t}\right)$ (d) $\frac{1}{a} f^1\left(\frac{t}{a}\right)$

(ix) The eigen values of Strum-Liouville boundary value problem are : 1

(a) real (b) complex

(c) imaginary (d) none of these

(x) If n is positive, then the value of $J_{-n}(x)$ is : 1

(a) $(-x)J_n(x)$ (b) $(-1)^n J_n(x)$

(c) $J_n(x)$ (d) $x^n J_n(x)$

UNIT—I

2. (a) Show that $L[t^n] = \frac{n!}{s^{n+1}}, s > 0$ and $L[e^{at}] = \frac{1}{s-a}, s > a$. 2+2

(b) Find $L[3t^2 - 2e^t + \sinh 3t + 5 \cos 4t]$. 3

(c) Evaluate $\int_0^\infty e^{-2t} \sin^3 t \, dt$. 3

3. (p) If f(x) is sectionally continous in every finite interval $0 \le x \le N$ is of exponential order 'a' for x > N, then prove that Laplace transform of f(x) exists for all s > a. 4

(q) Find Laplace Transform of $e^{-3t}(2 \cos 5t - 3 \sin 5t)$. 3

(r) Find Laplace Transform of $\int_0^t e^{-t} \cos t \, dt$. 3

UNIT—II

4. (a) State and prove change of scale property for inverse Laplace transform. 4

(b) Show that $L^{-1}\left[\frac{1}{(s^2+a^2)^2}\right] = \frac{1}{2a^3}(\sin at - at \cos at)$. 3

(c) Obtain inverse Laplace transform of $\frac{s^2-6}{s^3+4s^2+3s}$. 3

5. (p) If $L^{-1}[F(s)] = f(t)$, then show that $L^{-1}[F(s-a)] = e^{at}f(t)$. 4

(q) Use convolution theorem to evaluate $L^{-1}\left[\frac{1}{S(s^2+4)}\right]$. 3

(r) Find the inverse Laplace transform of $\frac{1}{(s-2)^3}$. 3

UNIT—III

6. (a) Solve the differential equation by Laplace transform method $\frac{d^2y}{dt^2} + 2\frac{dy}{dt} + y = 3te^{-t}$, $y(0) = 4$, $y'(0) = 2$. 5

(b) Using Laplace transform, solve the system of equations :

$$\frac{dx}{dt} = x - 2y; \quad \frac{dy}{dt} = -2x + y; \quad x(0) = 1 \; ; \; y(0) = 2.$$ 5

7. (p) Find the bounded solution of :

$u_x = 2u_t + u$, $u(x,0) = e^{-3x}$ for $x > 0$, $t > 0$. 5

(q) Let $u(x, t)$ be defined for $a \le x \le b$, $t > 0$. Compute Laplace transforms for u_x, u_t, u_{xx} and u_{tt}. 5

UNIT—IV

8. (a) Expand $f(x) = |x|$ in a Fourier series in $(-\pi, \pi)$. 5

(b) Find the Fourier series expansion of

$$f(x) = \begin{cases} 1 + \frac{2x}{\pi}, & -\pi < x < 0 \\ 1 - \frac{2x}{\pi}, & 0 < x < \pi \end{cases}$$ 5

9. (p) Obtain Fourier sine series for the function $f(x) = x^2$, $0 < x < 2$. 5

(q) If the trigonometric series $a_0 + \sum_{n=1}^{\infty}\left(a_n \cos\frac{n\pi x}{L} + b_n \sin\frac{n\pi x}{L}\right)$ converges uniformly to f(x) in $-L \le x \le L$, then it is a Fourier series of f(x). Obtain a_0, a_n and b_n. 5

UNIT—V

10. (a) Show that $P_n(x)$ is the co-efficient of h^n in the expression of $(1 - 2hx + h^2)^{-½}$. 4

(b) Prove that $J_{-½}(x) = \sqrt{\frac{2}{\pi x}}\cos x$. 3

(c) Rearrage $xy'' + 3y' + \lambda xy = 0$ in sturm-Liouville equation. 3

11. (p) Prove that the eigen values of the sturm-Liouville problem are real. 4

(q) Show that $J_0' = -J_1$ and $J_2 - J_0 = 2J_0''$. 3

(r) Show that all the roots of $P_n(x) = 0$ are real and lie between –1 and 1. 3