AQ—694

B.Sc. (Part—II) Semester-IV Examination
4S : MATHEMATICS
Paper—VII
(Laplace Transform & Fourier Series)

Time—Three Hours] [Maximum Marks—60

N.B. :— (1) Question No. **1** is compulsory and attempt at once.

(2) Solve **one** question from each unit.

1. Choose the correct alternative :

(i) L (cosh at) is equal to :

(a) $\frac{a}{s^2+a^2}$ (b) $\frac{s}{s^2-a^2}$

(c) $\frac{a}{s^2-a^2}$ (d) $\frac{s}{s^2+a^2}$ 1

(ii) $L^{-1}\left[\frac{1}{s^{n+1}}\right]$ is equal to :

(a) $\frac{n!}{t^n}$ (b) $\frac{t^n}{(n+1)!}$

(c) $\frac{t^n}{n!}$ (d) $\frac{(n+1)!}{t^n}$ 1

(iii) If an odd function f is expanded in a Fourier series over $[-\pi, \pi]$ then :

(a) $b_n = \frac{2}{\pi}\int_0^{\pi} f(x)\sin nx\,dx, a_n =$

(b) $a_n = 0, b_n = 0, \quad f(x) \neq 0$

(c) $a_n = \frac{2}{\pi}\int_0^{\pi} f(x)\cos nx\,dx\,,\ b_n = 0$

(d) $b_n = \frac{2}{\pi}\int_0^{l} f(x).\cos\frac{n\pi x}{\ell}\,dx$ 1

(iv) The period of the function sin x is :

(a) 2π (b) T

(c) $\frac{2\pi}{T}$ (d) 0

(v) If $L^{-1}[F(s)] = f(t)$ and $f(0) = 0$, then $L^{-1}[SF(s)]$ is :

(a) $\frac{f(t)}{t}$ (b) $tf(t)$

(c) $f'(t)$ (d) $-f'(t)$ 1

9. (p) Find the Fourier Series of :

$$f(x) = -x-1; \quad -1 \le x < 0$$
$$= x \quad ; \quad 0 \le x \le 1 .$$ 5

(q) Find the Fourier Series expansion of :

$$f(x) = \begin{cases} 1+\frac{2x}{\pi}, & -\pi < x < 0 \\ 1-\frac{2x}{\pi}, & 0 < x < \pi \end{cases}$$ 5

UNIT—V

10. (a) Prove that $xJ'_p = pJ_p - x\,J_{p+1}$. 4

(b) Using Rodrigues formula, find $p_n(x)$ for n = 0, 1, 2, 3. 3

(c) Evaluate $\int_a^b J_0(x)J_1(x)\,dx$. 3

11. (p) Show that DE_s of Bessel and Legendre are SL type. 4

(q) Prove that $J_{3/2}(x) = \sqrt{\frac{2}{\pi x}}\left(\frac{1}{x}\sin x - \cos x\right)$. 3

(r) Using Rodrigues formula, prove that :

$$\int_{-1}^{1} x^3 p_3(x)dx = \frac{4}{35}.$$ 3

UNIT—III

6. (a) Solve the differential equation by Laplace transform method :

$$\frac{dx}{dt} - y = e^t,\ \frac{dy}{dt} + x = \sin t,\ x=1,\ y=0 \text{ at } t=0.$$

5

(b) Solve the boundary value problem :

$u_t - ku_{xx}$, $u(x, 0) = \sin \pi x$, $u(0, t) = 0$, $u(1, t) = 0$, $0 < x < 1$, $t > 0$. 5

7. (p) Solve the integral equation :

$$f(x) = x + \int_0^x f(u) \cdot \sin(x - u) \cdot du.$$ 5

(q) Using Laplace transform, solve the simultaneous differential equations :

$$2\frac{dx}{dt} + y = 3x,\ \frac{dy}{dt} = -y - 2x$$

where $x(0) = 1$ and $y(0) = 2$. 5

UNIT—IV

8. (a) Obtain the Fourier Series for e^x in $(-\pi, \pi)$. 5

(b) Obtain the Fourier Series for $\sqrt{1 - \cos x}$ in $(0, 2\pi)$.

Show that $\frac{1}{2} = \sum_{n=1}^{\infty} \frac{1}{4n^2 - 1}$. 4+1

(vi) If $L[f(t)] = s^2$, then $L[f(at)]$ is :

$\frac{s^2}{a}$ (b) $\frac{s^2}{a^2}$

$\frac{s^2}{a^3}$ (d) $\frac{s^2}{a^4}$ 1

(vii) A function $f(x) = \begin{cases} x, & x \geq 0 \\ -x, & x < 0 \end{cases}$; is :

(a) Odd

(b) Even

(c) Even and Odd

(d) None of these 1

(viii) Half range Fourier Cosine Series is :

(a) $\frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx$

(b) $\frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \sin nx$

(c) $\sum_{n=1}^{\infty} a_n \cos nx$

(d) $\sum_{n=1}^{\infty} \cos nx$ 1

(ix) The D.E. $x^2y'' + xy' + (x^2 - n^2) y = 0$ is Sturm-Liauville if :

(a) $r(x) = 1,\ q(x) = 0,\ p(x) = 1$

(b) $r(x) = 1,\ q(x) = 1,\ p(x) = 0$

(c) $r(x) = x,\ q(x) = x,\ p(x) = -\frac{1}{x}$

(d) $r(x) = \frac{1}{x},\ q(x) = +x,\ p(x) = x$ 1

(x) The value of $J_{-\frac{1}{2}}(x)$ is equal to :

(a) $\sqrt{\frac{2}{\pi x}} \sin x$

(b) $\sqrt{\frac{\pi x}{2}} \sin x$

(c) $\sqrt{\frac{\pi x}{2}} \cos x$

(d) $\sqrt{\frac{2}{\pi x}} \cos x$ 1

UNIT—I

2. (a) If $L[f(t)] = F(s)$, then prove that :

$$L[t^n f(t)] = (-1)^n \frac{d^n}{ds^n} F(s).$$ 4

(b) Using Laplace transform of derivatives, prove that :

$$L[t \cos at] = \frac{s^2 - a^2}{(s^2 + a^2)^2}.$$ 3

(c) Find Laplace transform of $e^{-3t}(2 \cos 5t - 3 \sin 5t)$. 3

3. (p) If $L[f(t)] = F(s)$, then $L[f(at)] = \frac{1}{a} F\left(\frac{s}{a}\right)$. 4

(q) Find Laplace transform of $\sin t \cos 2t \cos 3t$. 3

(r) If $L\left[2\sqrt{\frac{t}{\pi}}\right] = \frac{1}{s^{3/2}}$, show that $L\left[\frac{1}{\sqrt{\pi t}}\right] = \frac{1}{\sqrt{s}}$. 3

UNIT—II

4. (a) State and prove the convolution theorem for inverse Laplace transform. 5

(b) Find the inverse Laplace transform of $\frac{1}{(s-2)(s+2)^2}$ by convolution theorem. 5

5. (p) If $L[f(t)] = F(s)$, then show that :

$$L\left[\int_0^t f(t)dt\right] = \frac{1}{s} F(s).$$ 5

(q) Find the inverse Laplace transform of $\frac{s^2}{(s^2 + a^2)(s^2 + b^2)}$ by convolution theorem. 5

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