AT–350

B.Sc. (Part-II) Semester-IV Examination

MATHEMATICS (NEW)

(Classical Mechanics)

Paper—VIII

Time : Three Hours] [Maximum Marks : 60

Note :— Question No. **1** is compulsory and attempt it once only and solve **ONE** question from each unit.

1. Choose the correct alternative (**1** mark each) : 10

(1) The virtual work on a mechanical system by the applied forces and reversed effective forces is :

(a) Zero (b) One

(c) Negative (d) None of these

(2) If q_i is cyclic, then $\frac{\partial H}{\partial q_i}=$

(a) 0 (b) 1

(c) –1 (d) None of these

(3) A particle moving in space has ______ degrees of freedom.

(a) One (b) Two

(c) Three (d) Four

(4) A cyclic co-ordinate will be ______ in Hamiltonian.

(a) Present (b) Absent

(c) Appear (d) None of these

(5) In a central force field, the angular momentum of a particle remains :

(a) Imaginary (b) Zero

(c) Real (d) Constant

(6) For a particle moving under a central force such that $V = Kr^{n+1}$, the virial theorem reduces to :

(a) $2\overline{T} = -n\overline{V}$ (b) $2\overline{T} = (n+1)\overline{V}$

(c) $2\overline{T} = \overline{V}$ (d) $2\overline{T} = -(n+1)\overline{V}$

(7) A stationary point of the function f(x) includes ______.

(a) a maximum point (b) a minimum point

(c) a point of inflection (d) all of these

(8) Two curves which are close in the sense of 3^{rd} order proximity necessarily not be close in the sense of ______ order proximity.

(a) 0^{th} (b) 1^{st}

(c) 2^{nd} (d) 4^{th}

(9) The general displacement of a rigid body with ______ point fixed is a rotation about some axis.

(a) One (b) Two

(c) Three (d) None of these

(10) ______ rotation do not commute.

(a) Infinite (b) Finite

(c) Countable (d) None of these

UNIT—I

2. (a) Derive the Lagranges equations of motion for conservative system from D'Alemberts principle. 6

(b) Find the equations of motion for a particle moving in space by using Cartesian coordinate. 4

3. (p) Construct a Lagrangian for a spherical pendulum and then obtain the Lagrange's equations of motion. 5

(q) Show that the shortest distance between two points in a plane is a straight line. 5

UNIT—II

4. (a) State and prove the virial theorem of the system. 1+4

(b) Derive the differential equation for the orbit of a particle in a central force field. 5

5. (p) Show that if a particle describes a circular orbit under the influence of an attractive central force directed towards point on the circle then the force varies as the inverse fifth power of the distance. 5

(q) Derive the equation of a path of a particle in a central force field in the form :

$$\phi = \phi_0 + \left(\frac{h}{m}\right) \int_{r_0}^{r} \frac{dr}{fr^2}.$$ 5

UNIT—III

6. (a) Prove that the functional $I[y(x)] = \int_{x_1}^{x_2} F(x, y, y')\,dx$ where the end points are fixed, is extremum if y satisfies the differential equation $F_y - \frac{d}{dx} F_{y'} = 0$. 5

(b) Define N^{th} order distance between curve. Find the distance between the curves :
$y(x) = x\,e^{-x}$, $y_1(x) = 0$ on [0, 2]. 1+4

7. (p) Show that the functional $I[y(x)] = \int_0^1 \{2y(x) + y'(x)\}\,dx$ defined in the space $c_1[0, 1]$ is continuous on the function $y_0(x) = x$ in the sense of first order proximity. 5

(q) Find the extremals of the functional $I[y] = \int_0^{2\pi} (y'^2 - y^2)\,dx$ that satisfies the boundary conditions $y(0) = 1$, $y(2\pi) = 1$. 5

UNIT—IV

8. (a) State and prove least action principle. 5

(b) State Hamilton's principle. Prove that :

$$\frac{dH}{dt} = \frac{\partial H}{\partial t} = -\frac{\partial L}{\partial t}.$$ 5

9. (p) Prove that A cyclic co-ordinate will not occur in the Routhian R. 5

(q) Use Hamilton's principle to find the equations of motion of a particle of mass moving in space in a conservative force field $\vec{F}$. 5

UNIT—V

10. (a) State and prove Euler's theorem. 6

(b) Define infinitesimal rotation. Show that infinitesimal rotations commute. 4

11. (p) Prove that :

(i) If $A = I + \epsilon$, then the inverse rotation matrix $A^{-1} = I - \epsilon$. 3

(ii) Infinitesimal rotation matrix ϵ is antisymmetric. 3

(q) Prove that a rotation matrix A is orthogonal. 4