AU–105

B.Sc. Part—II (Semester—IV) Examination
MATHEMATICS
Paper—VIII
(Classical Mechanics)

Time : Three Hours] [Maximum Marks : 60

Note :— (1) Question No. **1** is compulsory and attempt it once only.

(2) Solve ***one*** question from each unit.

1. Choose the correct alternative :

(i) If the equation of constraint varies with time, then it is called as :

(a) Holonomic constraint

(b) Stationary or scleronomous constraint

(c) Moving or Rheonomous constraint

(d) None of these 1

(ii) The polar equation of a conic section is

$$\frac{\ell}{r} = 1 + e\cos(\theta - \theta_0)$$

where ℓ is its semi lotus rectum and e is eccentricity.

If e < 1, then conic represents ______.

(a) Hyperbola (b) Parabola

(c) Circle (d) Ellipse 1

(iii) If the function f(x) has maximum or minimum value at some point $x = x_0$, then the point $x = x_0$ is called as ______.

(a) Stationary point (b) Critical point

(c) Extremum point (d) None of these 1

(iv) The shortest distance between two points in a space is _____.

(a) A circle (b) A straight line

(c) An ellipse (d) A parabola 1

(v) Functions $y(x)$ for which $\delta I[y(x)] = 0$ are called ______.

(a) Admissible functions (b) Absolute functions

(c) Stationary functions (d) None of these 1

(vi) H is the Hamiltonian of the system then a generalized coordinate q_i is said to be cyclic if _______.

(a) $\frac{\partial H}{\partial q_i} \neq 0$ (b) $\frac{\partial H}{\partial q_i} > 0$

(c) $\frac{\partial H}{\partial q_i} = 0$ (d) $\frac{\partial H}{\partial q_i} < 0$ 1

(vii) If a 3×3 matrix A is a rotation matrix, then A is orthogonal and _____.

(a) $|A| = 0$ (b) $|A| \neq 1$

(c) $|A| = 1$ (d) None of these 1

(viii) The number of degrees of freedom for a motion of a particle along a straight line are _____.

(a) 0 (b) 1

(c) 2 (d) 3 1

(ix) If H is the Hamiltonian of the system and $p_i = \frac{\partial L}{\partial \dot{q}_i}$ is the generalized momentum associated with generalized coordinate q_i, then the Hamilton's equations are _____.

(a) $\frac{\partial H}{\partial p_i} = \dot{q}_i$, $\frac{\partial H}{\partial q_i} = \dot{p}_i$ (b) $\frac{\partial H}{\partial p_i} = -\dot{q}_i$, $\frac{\partial H}{\partial q_i} = \dot{p}_i$

(c) $\frac{\partial H}{\partial p_i} = \dot{q}_i$, $\frac{\partial H}{\partial q_i} = -\dot{p}_i$ (d) $\frac{\partial H}{\partial p_i} = -\dot{q}_i$, $\frac{\partial H}{\partial q_i} = -\dot{p}_i$ 1

(x) For a particle moving under a central force such that $V = kr^{n+1}$, the virial theorem reduces to _____.

(a) $\overline{T} = (n+1)\overline{V}$ (b) $\overline{T} = (n-1)\overline{V}$

(c) $2\overline{T} = (n-1)\overline{V}$ (d) $2\overline{T} = (n+1)\overline{V}$ 1

UNIT—I

2. (a) Derive the Lagrange's equations of motion in the form $\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_i}\right)-\frac{\partial L}{\partial q_i}=0, i=1, 2, \ldots n$ for conservative system from D'Alembert's principle. 6

(b) Construct a Lagrangian for a spherical pendulum and obtain the Lagrange's equations of motion. 4

OR

3. (p) Use D'Alembert's principle to obtain the equations of motion of a simple pendulum. 5

(q) Two particles of masses m_1 and m_2 are connected by a light inextensible string which passes over a small smooth fixed pulley. If $m_1 > m_2$, then show that the common acceleration of the particles is $\frac{(m_1-m_2)}{(m_1+m_g)}g$. 5

UNIT—II

4. (a) State and prove Virial theorem. 1+4

(b) Show that if a particle describes a circular orbit under the influence of an attractive central force directed towards point on the circle then the force varies as the inverse fifth power of the distance. 5

OR

5. (p) Prove that in a central force field, the areal velocity is conserved. 5

(q) Prove that if the potential energy is a homogeneous function of degree -1 in the radius vector $\bar{r}_i$, then the motion of a conservative system takes place in a finite region of space only if the total energy is negative. 5

UNIT—III

6. (a) Find the extremals of $I[y(x)] = \int_a^b [y^2 + y'^2 + 2ye^x]dx$. 5

(b) Show that the functional :

$$I[y(x)] = \int_0^1 [2y(x) + y'(x)]dx$$

defined in the space $C_1[0, 1]$ is continuous on the function $y_0(x) = x$ in the sense of first order proximity. 5

OR

7. (p) Prove that if x does not occur explicitly in F then, $F_{y'}y' - F$ = constant. 5

(q) Find the extremals of the functional :

$$I[y(x)] = \int_0^{\log 2} (e^{-x}y'^2 - e^x y^2)dx \quad 5$$

UNIT—IV

8. (a) Obtain the Hamiltonian and then deduce the equations of motion for a simple pendulum. Show that the Hamiltonian of the system is the total energy and also the constant of motion. 6

(b) A particle moves on a smooth surface under gravity. Use Hamilton's principle to show that the equations of motion are :

$$\ddot{x} = \ddot{y} = 0, \ \ddot{z} = -g$$

where the vertical is taken along the z-axis. 4

OR

9. (p) Define : Hamiltonian H. Derive the Hamilton's equations or the canonical equations of Hamilton. 1+4

(q) Use Hamilton's principle to find the equations of motion of a particle of mass moving in space in a conservative force field F. 5

UNIT—V

10. (a) Describe the frame rotation and obtain the rotation matrix. 5

(b) If $A_1 = I + \epsilon_1$ and $A_2 = I + \epsilon_2$ be two infinitesimal rotations, then prove that infinitesimal rotations commute. 5

OR

11. (p) Prove that if A is any 2×2 orthogonal matrix with determinant $|A| = 1$, then A is a rotation matrix. 5

(q) If $A = I + \epsilon$, then prove that the inverse rotation matrix is $A^{-1} = I - \epsilon$. 5