AS–1426

B.Sc. (Part—II) Semester—IV Examination
MATHEMATICS (New)
(Classical Mechanics)
Paper—VIII

Time : Three Hours] [Maximum Marks : 60

Note :— Question No. **1** is compulsory and attempt it once only and solve **ONE** question from each unit.

1. Choose the correct alternative : (1 mark each) :— 10

(i) Each planet describes ____________ having the sun in one of its foci.

(a) An ellipse (b) A circle

(c) A hyperbola (d) None of these

(ii) In a central force field, the areal velocity is ______________ .

(a) Not constant (b) Not conserved

(c) Conserved (d) None of these

(iii) The maximum point and the minimum point of a function f(x) are called the ________ .

(a) Extremum (b) Functional

(c) Continuity of a functional (d) None of these

(iv) If two curves are closed in the sense of k^{th} order proximity, then they are close in the sense of _____________ order proximity.

(a) Larger (b) Smaller

(c) Equal (d) None of these

(v) Hamilton's Equation is q_i = __________ .

(a) $\frac{\partial H}{\partial P_i}$ (b) $\frac{\partial H}{\partial q_i}$

(c) $\frac{\partial H}{\partial t}$ (d) None of these

(vi) If a generalised co-ordinate does not appear in H_1 then the corresponding conjugate momentum is ________ .

(a) Conserved (b) Not conserved

(c) Not constant (d) None of these

(vii) The shortest distance between two points in a plane is ________ .

(a) A straight line (b) An ellipse

(c) A parabola (d) A circle

(viii) If q_i are the generalised coordinates and the constraints are holonomic, then δq_i are ________ .

(a) Zero (b) Equivalent

(c) Dependent (d) Independent

(ix) The sum of the finite rotation depends on the ________ of the rotation.

(a) Degree (b) Order

(c) Degree and order (d) None of these

(x) The general displacement of a rigid body with ________ point fixed is a rotation about some axis.

(a) One (b) Two

(c) Three (d) None of these

UNIT—I

2. (a) Show that the shortest distance between two points in a plane is a stright line. 5

(b) Find the Lagrangian for the system consisting of a simple pendulum of mass m_2, with mass m_1 at the point of support which can move on a horizontal line lying in the plane in which m_2 moves. 5

3. (p) Two particles of masses m_1 and m_2 are connected by a light inextensible string which passes over a small smooth fixed pulley. If $m_1 > m_2$, then show that the common accleration of particles is $(m_1 - m_2)g/(m_1 + m_2)$. 5

(q) State and prove D'Alemberts principle. 5

UNIT—II

4. (a) Derive the differential equation for the orbit of a particle in a central force field. 5

(b) Prove that the square of the periodic time of the planet is proportional to the cube of the major axis of its orbit. 5

5. (p) Prove that in a central force field, the areal velocity is conserved. 5

(q) A particle moves on a curve $r^n = a^n \cos n\theta$ under the influence of a central force field. Find the law of force. 5

UNIT—III

6. (a) Show that the functional :

$$I[y(x)] = \int_0^1 x^3 \sqrt{1+y^2(x)}\, dx.$$

defined on the set of functions $y(x) \in c\,[0, 1]$ is continuous on the function $y_0(x) = x^2$ in the sense of zeroth order proximity. 5

(b) Find the extremal of the functional

$$I[y(x)] = \int_{-1}^{0} (480y - y'''^2)\, dx.$$

$$y(0) = 0,\ y(-1) = \frac{1}{3},\ y'(0) = 0,\ y'(-1) = -2,\ y''(0) = 0,\ y''(-1) = 8.$$ 5

7. (p) Prove that if x does not occur explicitly in F, then $F_{y'}\, y' - F$ = constant. 6

(q) Find the distance between the curves

$y(x) = xe^{-x}$, $y_1(x) = 0$ on $[0, 2]$. 4

UNIT—IV

8. (a) State the Hamilton's principle. Prove that Hamilton principle is a necessary and sufficient condition for Lagranges equations. 5

(b) Discuss the Routh's procedure. 5

9. (p) (i) Prove that $\frac{dH}{dt} = \frac{\partial H}{\partial t} = \frac{-\partial L}{\partial t}$. 3

(ii) Prove that A cyclic co-ordinate will be absent in Hamiltonian. 3

(q) Give the physical significance of H. 4

UNIT—V

10. (a) Define Infinitisimal rotation. Prove that if $A = I + \in$, then the inverse rotation matrix $A^{-1} = I - \in$. 4

(b) Prove that the general displacement of a rigid body with one point fixed is a rotation about some axis. 6

11. (p) Define Eulerian Angle. 2

Prove that the change in the components of a vector under the infinitesimal transformation of the coordinate system is given by

$$d\vec{r} = \vec{r} \times d\vec{u}\cdot$$ 4

(q) Show that the two complex eigen values of an orthogonal matrix representing a proper rotation are $e^{\pm i\phi}$, where ϕ is the angle of rotation. 4