B.Sc. Part-II (Semester-IV) Examination

MATHEMATICS

(Classical Mechanics)

Paper—VIII

Time : Three Hours] [Maximum Marks : 60

Note :— (1) Question No. **1** is compulsory and attempt it once only.

(2) Solve **ONE** question from each unit.

1. Choose the correct alternative :

(i) Each planet describes ______ having the sun at one of its foci. 1

(a) An ellipse (b) A circle

(c) A hyperbola (d) None of these

(ii) If a bead is sliding along the wire then the constraint is ______. 1

(a) Holonomic (b) Non-holonomic

(c) Superfluous (d) None of these

(iii) For an inverse square law, the virial theorem reduces to ______. 1

(a) $2\overline{T} = -n\overline{V}$ (b) $2\overline{T} = n\overline{V}$

(c) $2\overline{T} = \overline{V}$ (d) $2\overline{T} = -\overline{V}$

(iv) The virtual work on a mechanical system by the applied forces and reversed effective forces is ______. 1

(a) Zero (b) One

(c) Negative (d) None of these

(v) The shortest distance between two points in a space is ______. 1

(a) A circle (b) A straight line

(c) An ellipse (d) A parabola

(vi) If H is the Hamiltonian of the system then a generalized coordinate q_i is said to be cyclic if ______. 1

(a) $\frac{\partial H}{\partial q_i} \neq 0$ (b) $\frac{\partial H}{\partial q_i} > 0$

(c) $\frac{\partial H}{\partial q_i} = 0$ (d) $\frac{\partial H}{\partial q_i} < 0$

(vii) A square matrix A is said to be orthogonal if ______. 1

(a) $A = A^T$ (b) $A^T = A^{-1}$

(c) $A = A^{-1}$ (d) None of these

(viii) The general displacement of a rigid body with ______ point fixed is a rotation about some axis. 1

(a) One (b) Two

(c) Three (d) None of these

(ix) The sum of the finite rotations depends on the ______ of the rotation. 1

(a) Degree (b) Order

(c) Both Degree and Order (d) None of these

(x) A particle moving in a space has ______ degrees of freedom. 1

(a) One (b) Two

(c) Three (d) Four

UNIT—I

2. (a) Derive the Lagrange's equations of motion in the form :

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}}\right) - \frac{\partial L}{\partial q_i} = 0, i = 1, 2, \ldots\ldots, n.$$

for conservative system from D'Alembert's principle. 6

(b) A bead is sliding on a uniformly rotating wire in a force-free space, then show that the acceleration of the bead is $\ddot{r} = rw^2$, where w is the angular velocity of rotation. 4

3. (p) Two particles of masses m_1 and m_2 are connected by a light inextensible string which passes over a small smooth fixed pulley. If $m_1 > m_2$, then show that the common acceleration of the particles is $\left\{\frac{(m_1 - m_2)}{(m_1 + m_2)}\right\}g$. 5

(q) Obtain the equations of motion of a simple pendulum by using D'Alembert's principle. 5

UNIT—II

4. (a) For a central force field, show that Kepler's second law is a consequence of the conservation of angular momentum. 5

(b) Prove that if the potential energy is a homogeneous function of degree –1 in the radius vector $\vec{r}_i$, then the motion of a conservative system takes place in a finite region of space only if the total energy is negative. 5

5. (p) Prove that in a central force field the areal velocity is conserved. 5

(q) Show that if a particle describes a circular orbit under the influence of an attractive central force directed towards point on the circle, then the force varies as the inverse fifth power of the distance. 5

UNIT—III

6. (a) Show that the functional :

$$I[y(x)] = \int_0^1 \{2y(x) + y'(x)\}\, dx$$

defined in the space $c_1[0, 1]$ is continuous on the function $y_0(x) = x$ in the sense of first order proximity. 5

(b) Find the extremals of $I[y(x)] = \int_a^b [y^2 + y'^2 + 2ye^x]dx$. 5

7. (p) Find the extremals of the functional :

$$I[y(x)] = \int_a^b [16y^2 - y''^2 + x^2]dx. \quad 5$$

(q) Write down the Euler-Ostrogradsky equation for the functional :

$$I[z(x, y)] = \iint_D \left\{\left(\frac{\partial z}{\partial x}\right)^4 + \left(\frac{\partial z}{\partial y}\right)^4 + 12zf(x, y)\right\} dx\, dy. \quad 5$$

UNIT—IV

8. (a) Show that Hamilton's principle can be derived from D'Alembert's principle. 5

(b) Define Hamiltonian H. Derive the Hamilton's equations for the Hamiltonian H of the system. 1+4

9. (p) Deduce the Hamilton's equations of motion of a particle of mass m in Cartesian coordinates (x, y, z). 5

(q) Define Routhian, prove that a cyclic coordinate will not occur in the Routhian R. 1+4

UNIT—V

10. (a) Prove that if A is any 2 × 2 orthogonal matrix with determinant | A | = 1, then A is a rotation matrix. 5

(b) Define infinitesimal rotation. Prove that infinitesimal rotations commute. 1+4

11. (p) Show that two complex eigenvalues of an orthogonal matrix representing a proper rotation are $e^{\pm i\phi}$, where ϕ is the angle of rotation. 5

(q) Prove that the general displacement of a rigid body with one point fixed is a rotation about some axis. 5