AV–1760

B.Sc. (Part—II) Semester-IV Examination

MATHEMATICS

(Classical Mechanics)

Paper—VIII

Time : Three Hours] [Maximum Marks : 60

Note :— Question No. **1** is compulsory and attempt it once only and solve **ONE** question from each unit.

1. Choose the correct alternative (**1** mark each) : 10

(i) For an inverse square law, the virial theorem reduces to ______.

(a) $2\overline{T} = -n\overline{V}$ (b) $2\overline{T} = n\overline{V}$

(c) $2\overline{T} = \overline{V}$ (d) $2\overline{T} = -\overline{V}$

(ii) The shortest distance between two points in space is ______.

(a) A straight line (b) An ellipse

(c) A parabola (d) A circle

(iii) A bead sliding along the wire. The constraint is ______.

(a) Holonomic (b) Non-holonomic

(c) Superfluous (d) None of these

(iv) The square of the periodic time of the planet is proportional to the ______ of the major axis of its orbit.

(a) Square (b) Cube

(c) Not both (a) and (b) (d) None of these

(v) A variable quantity whose value is determined by one or more than one function is called ______.

(a) An extremum (b) A point of inflection

(c) A functional (d) None of these

(vi) The founder of the calculus of variations is ______.

(a) Lagrange (b) Leibnitz

(c) J. Bernoulli (d) Euler

(vii) If q_i is cyclic, then $\frac{\partial H}{\partial q_i}$ = ______.

(a) 1 (b) –1

(c) 0 (d) None of these

(viii) For a single particle system, the least action principle yield ______.

(a) $\Delta \int \sqrt{2m(H-V)}\, ds = 0$ (b) $\Delta \int \sqrt{2m(H+V)}\, ds = 0$

(c) $\Delta \int \sqrt{m(H-V)}\, ds = 0$ (d) None of these

(ix) A finite rotation can not be represented by ______.

(a) Double vector (b) Triple vector

(c) A single vector (d) None of these

(x) Infinitesimal rotation holds ______.

(a) Commutativity (b) Not Commutativity

(c) Distributivity (d) None of these

UNIT—I

2. (a) Prove virtual work on a mechanical system (for which the net virtual work of the forces of constraint vanishes) by the applied forces and the reversed effective forces is zero. 5

(b) Derive the Lagrange's equation of motion in the form $\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_i}\right) - \frac{\partial L}{\partial q_i} = Q_i'$ for a system which is partly conservative. 5

3. (p) Discuss the motion of a particle in a plane by using polar coordinates. 5

(q) If L is a Lagrangian for a system of 'n' degree of freedom satisfying Lagrange's equations, show by direct substitution that $L' = L + \frac{dF}{dt}$, $F = F(q_1, \ldots\ldots q_n t)$ also satisfies Lagrange's equations where F is any arbitrary but differentiable function of its argument. 5

UNIT—II

4. (a) Prove for a central force field F, the path of a particle of mass m is given by

$$\frac{d^2u}{d\theta^2} + u = -\frac{m}{h^2u^2} F\left(\frac{1}{u}\right),\ u = \frac{1}{r}.$$ 5

(b) Prove that for a particle moving under a central force such that $V = kr^{n+1}$, the virial theorem reduces to $2\bar{T} = (n+1)\bar{V}$. 5

5. (p) Prove the following relations :

(i) $t = \int_{r_0}^{r} \frac{dr}{f}$

(ii) $\phi = \phi_0 + \left(\frac{h}{m}\right) \int_0^t \frac{dt}{r^2}$. 3+3

(q) Prove that in a central force field, the areal velocity is conserved. 4

UNIT—III

6. (a) Find the extremals of the functional :

$$I[y(x)] = \int_0^{\log 2} (e^{-x}y'^2 - e^x y^2)\,dx.$$ 5

(b) Find the shortest curve joining the points (x_1, y_1) and (x_2, y_2) in a plane. 5

7. (p) Define the n^{th} order distance. Find the second order distance between the curves $y = -\cos x$ and $y_1 = x$ on $[0, \pi/3]$. 1+4

(q) Prove that the functional $I[y(x)] = \int_{x_1}^{x_2} F(x, y, y')\,dx$ where the end points are fixed, is extremum if y satisfies the differential equation $F_y - \frac{d}{dx}Fy' = 0$. 5

UNIT—IV

8. (a) Obtain Hamilton Equations. Prove that if a generalised co-ordinate does not appear in H, then the corresponding conjugate momentum is conserved. 2+2

(b) Derive Lagrange's equations for nonholonomic conservative system. 6

9. (p) Derive the Hamilton's equations from variational principle. 5

(q) Construct the Routhian in spherical polar coordinates for a particle moving in space under the action of a conservative force field. 5

UNIT—V

10. (a) Prove that the change in the components of a vector under the infinitesimal transformation of the coordinate system is given by $d\vec{r} = \vec{r} \times d\vec{u}$. 5

(b) If A is any 2 × 2 orthogonal matrix with determinant | A | = 1, then prove that A is a rotation matrix. 5

11. (p) Define infinitesimal rotation. Prove that Infinitesimal rotation matrix $\in$ is antisymmetric. 5

(q) Show that the angle of rotation ϕ is given in terms of Eulerian angles by :

$$\cos\frac{\phi}{2} = \cos\frac{\theta}{2}\cdot\cos\frac{1}{2}(\phi + \psi).$$ 5