AT–347

B.Sc. (Part–II) Semester—IV Examination
4S : MATHEMATICS (OLD)
(Laplace Transform and Fourier Series)
Paper—VII

Time : Three Hours] [Maximum Marks : 60

N.B. :— (1) Question No. **1** is compulsory and attempt at once.
(2) Solve **ONE** question from each unit.

1. Choose the correct alternative (**1 mark each**) :

(i) $L^{-1}\left[\frac{1}{s^3}\right]$ is equal to :

(a) x (b) $\frac{x^2}{2}$

(c) x^3 (d) 1

(ii) If $L^{-1}[F(s)] = f(x)$, then $L^{-1}[s \cdot F(s)]$ is equal to :

(a) $sf(x)$ (b) $f(xs)$

(c) $f'(x)$ (d) $xf(x)$

(iii) Laplace transform of the function $f(x) = 1$ is :

(a) $\frac{1}{s+1}$, $s > 0$ (b) s, $s > 1$

(c) $\frac{1}{s-1}$, $s > 1$ (d) $\frac{1}{s}$, $s > 0$

(iv) $\int_0^\infty \frac{\sin x}{x}\, dx$ is equal to :

(a) 0 (b) $\frac{\pi}{4}$

(c) $\frac{\pi}{2}$ (d) π

(v) $L[f(x)] = F(s)$, then $L[e^{ax}f(x)]$ is equal to :

(a) $F(s - a)$
(b) $F(s + a)$
(c) $F(a - s)$
(d) $F(s)$

(vi) The period of the sin x is :

(a) 2π
(b) T
(c) 0
(d) $\frac{2\pi}{T}$

(vii) The function $f(x) = -\sin x$ is :

(a) Even
(b) Odd
(c) Odd and even
(d) None of these

(viii) If $L[f(x)] = F(s)$, then prove that $L[x\, f(x)]$ is equal to :

(a) $-F'(s)$
(b) $F'(s)$
(c) $f(x)$
(d) $f'(x)$

(ix) The value of $J_{-1/2}^{(x)}$ is equal to :

(a) $\sqrt{\frac{2}{\pi x}} \sin x$
(b) $\sqrt{\frac{\pi x}{2}} \sin x$
(c) $\sqrt{\frac{\pi x}{2}} \cos x$
(d) $\sqrt{\frac{2}{\pi x}} \cos x$

(x) The value of $\frac{1}{2^n n!} \frac{d^n}{dx^n} (x^2 - 1)^n$ is :

(a) $J_n(x)$
(b) $J_{-n}(x)$
(c) $P_n(x)$
(d) $P_{-n}(x)$ 10

UNIT—I

2. (a) Find the Laplace transform of $\cosh^4 x$. 3

(b) If $L[f(x)] = F(s)$, then show that $L[x^n f(x)] = (-1)^n \frac{d^n}{ds^n} \cdot F(s)$. 4

(c) Prove that $\int_0^\infty \frac{e^{-at} - e^{-bt}}{t} \cdot dt = \log\left[\frac{b}{a}\right]$. 3

3\. (p) If $f(x) = \cosh 2x \cdot \cos 2x$, then show that :

$$L[f(x)] = \frac{s^3}{s^4 + 8^2}.$$ 5

(q) Using Laplace transform, prove that :

$$L[x \cos x] = \frac{s^2 - a^2}{(s^2 + a^2)^2}$$ 5

UNIT—II

4\. (a) State Convolution theorem, find $L^{-1}\left[\frac{1}{(s^2 + a^2)^2}\right]$ by using convolution theorem. 1+4

(b) Find the inverse Laplace transform of $\frac{3s + 2}{2s^2 - 4s + 3}$. 5

5\. (p) Find the inverse Laplace transform of $\frac{8}{(s^2 + 1)^3}$ by using convolution theorem. 5

(q) Find the inverse Laplace transform of $\frac{2s}{2s^2 - s - 5}$. 5

UNIT—III

6\. (a) Solve the D.E. $y'' + 4y' + 3y = e^{-t}$, where $y(0) = y'(0) = 1$. 5

(b) Solve the D.E's using Laplace transform $\frac{dx}{dt} - y = e^t$, $\frac{dy}{dt} + x = \sin x$, where $x(t) = 1$, $y(t) = 0$ at $t = 0$. 5

7\. (p) Solve the integral equation $f(x) = x + 2\int_0^x \cos(x - u)\, f(u)\, du$. 5

(q) Solve the D.E. $y'' - 2y' + 2y = 0$, $y(0) = y'(0) = 1$. 5

UNIT—IV

8. (a) Obtain the Fourier series for e^x in $[-\pi, \pi]$. 4

(b) If the Trigonometric series :

$$\frac{a_0}{2} + \sum_{n=1}^{\infty} (a_n \cos nx + b_n \sin nx)$$

converges uniformly to f(x) in $c \le x \le c + 2\pi$, then f(x) is the Fourier series, obtain a_0, a_n and b_n. 6

9. (p) Obtain Fourier sine series for function $f(x) = x^2, 0 < x < 2$. 5

(q) Find the Fourier cosine series for the function $f(x) = \begin{cases} x & , 0 \le x \le 4 \\ 8 - x & , 4 \le x \le 8 \end{cases}$. 5

UNIT—V

10. (a) Prove that $p_n(x) = \frac{1}{2^n n!} \frac{d^n}{dx^n} (x^2 - 1)^n$. 5

(b) Prove that the eigen values of the SL-problem are real. 5

11. (p) Prove that $J_{\frac{3}{2}}(x) = \sqrt{\frac{2}{\pi x}} \cdot \left[\frac{\sin x}{x} - \cos x\right]$. 5

(q) Prove that the recurrence formula :

$$xJ'_n = nJ_n - xJ_{n+1}.$$ 5