AV–1807

B.Sc. (Part—III) Semester-VI Examination

MATHEMATICS (OLD) (UPTO WINTER-2018)

(Graph Theory)

Paper—XII

Time : Three Hours] [Maximum Marks : 60

Note :— (1) Question No. **1** is compulsory and attempt it at once.

(2) Attempt **ONE** question from each unit.

1. Choose the correct alternative :

(i) Vertices with which a walk begins and ends are called ______.

(a) Terminal vertices (b) Isolated vertices

(c) Pendent vertices (d) None of these 1

(ii) The number of vertices of odd degree in a graph is always ___________.

(a) Even (b) Odd

(c) Even and odd (d) None of these 1

(iii) In any tree (with two or more vertices), there are at least _________ pendent vertices.

(a) 1 (b) 2

(c) 3 (d) 4 1

(iv) The length of the longest path in a tree is called its _________.

(a) Centre (b) Radius

(c) Diameter (d) Walk 1

(v) Number of edges in the smallest cut-set of a connected graph is called as _______.

(a) Vertex connectivity (b) Edge connectivity

(c) Separability (d) None of these 1

(vi) The formula $n-e+f = 2$ for planar graph is given by _________.

(a) Euler (b) Caley

(c) Kuratowski (d) Hamiltonian 1

(vii) The dot product of two vectors, one corresponding a subgraph g and the other g' is _______ if the number of edges common to g and g' is odd.

(a) Zero (b) One

(c) Two (d) Three 1

(viii) The set of all ______ in W_G forms a subspace W_Γ.

(a) Circuit vectors (b) Cut-set vectors

(c) Circuit and cut-set vertices (d) None of these 1

(ix) If B is a circuit matrix of a connected graph G with e edges and n vertices then rank of B is :

(a) e–n+1 (b) e+n–1

(c) n–1 (d) n+1 1

(x) In cut-set matrix column with all _______ corresponds to an edge forming a self-loop.

(a) Ones (b) Edges

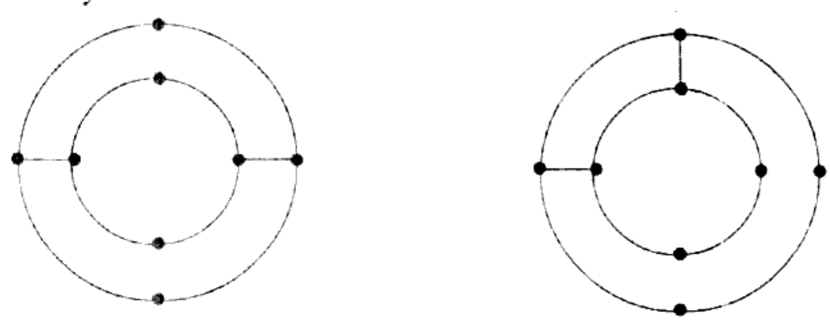
(c) Vertices (d) Zeros 1

UNIT—I

2. (a) Define Hamiltonian circuit. If n is an odd number ≥ 3 then prove that in a complete with n vertices there are (n–1)/2 edge-disjoint Hamiltonian circuit. 1+4

(b) Explain Konigsberg Bridge Problem. 5

3. (p) Define isomorphism between two graphs. Find whether the following graphs are isomorphic or not and why ?



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(q) Prove that a graph G is disconnected if and only if its vertex set V can be partitioned into two nonempty, disjoint subsets V_1 and V_2 such that there exist no edge in G whose one end vertex is in subset V_1 and the other in subset V_2. 5

UNIT—II

4. (a) Define minimally connected graph. Prove that a graph is a tree if and only if it is minimally connected. 1+4

(b) Define tree. If G is a graph with n vertices then prove that following statements are equivalent :

(i) G is a tree

(ii) G is connected and has n–1 edges. 1+4

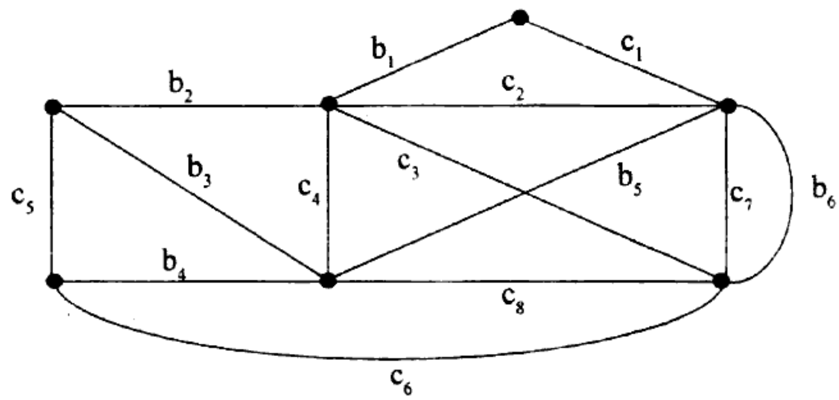
5. (p) Define centre of a tree and show that every tree has either one or two centers. 1+4

(q) Prove that for any connected graph with n-vertices, e-edges, its spanning tree has n–1 branches and e–n+1 chords. 5

UNIT—III

6. (a) Define edge connectivity and vertex connectivity. Prove that the vertex connectivity of any graph G can never exceed the edge connectivity of G. 5

(b) For the following graph G, find rank of G, nullity of G and fundamental circuits with reference to the spanning tree : T = $\{b_1, b_2, b_3, b_4, b_5, b_6\}$.



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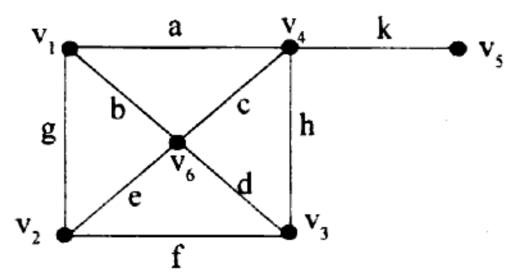
7. (p) Define cutset. List all the cut sets in the following graph :



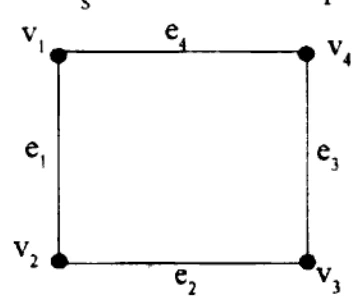
1+4

(q) Define planar graph. Show that Kuratowski's $K_{3,3}$ graph is non-planar. 1+4

UNIT—IV

8. (a) Prove that W_Γ of all circuits vectors including zero vector in W_G forms a subspace of W_G. 5

(b) Let G be a graph given in a figure. Find W_Γ, W_S, $W_\Gamma \cap W_S$ and $W_\Gamma \cup W_S$ where W_Γ is a circuit subspace and W_S is a cut-set subspace.



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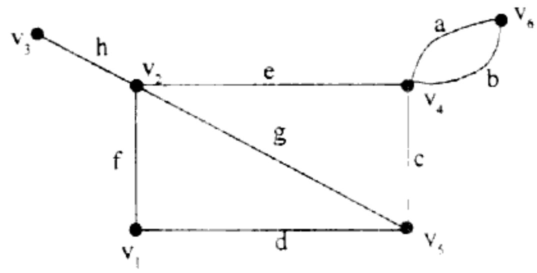
9. (p) Prove that the dimension of the cut-set subspace W_S is equal to the rank r of the graph and the number of cut-set vector (including O) in W_S is 2^r. 5

(q) In the vector space of a graph prove that the circuit space and cut-set subspace are orthogonal to each other. 5

UNIT—V

10. (a) Let A(G) be an incidence matrix of a connected graph G with n vertices. Prove that an (n–1) × (n–1) submatrix of A(G) is non singular if and only if n–1 edges corresponding to the n–1 columns of this matrix constitute a spanning tree in G. 5

(b) Find the path matrix $P(V_3, V_4)$ and circuit matrix B(G) of the following graph G.



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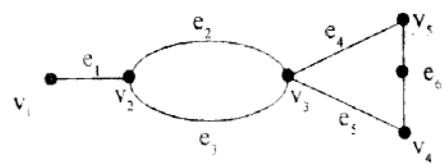
11. (p) Prove that if an incidence matrix of a connected graph G with n vertices then the rank of A(G) is n–1. 5

(q) Find incidence matrix A(G) and adjacency matrix X(G) for the graph.



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