AS–1400

B.Sc. Part—II (Semester—III) Examination
MATHEMATICS (UPTO S/17) (Old)
(Partial Differential Equations)
Paper—VI

Time : Three Hours] [Maximum Marks : 60

Note :— (1) Question No. 1 is compulsory.

(2) Solve **ONE** question from each Unit.

1. Choose the correct alternatives :

(i) The condition for the PDE f(x, y, z, p, q) = 0 and g(x, y, z, p, q) = 0 to be compatible is :

(a) $J_{xz} + J_{yz} + pJ_{xp} + qJ_{zq} = 0$

(b) $J_{xy} + pJ_{zp} + qJ_{zq} = 0$

(c) $J_{xp} + J_{yq} + pJ_{zp} + qJ_{zq} = 0$

(d) None of these

(ii) A PDE z = px + qy + F(p, q) has the :

(a) Charpit's form

(b) Lagrange's form

(c) Monge's form

(d) Clairaut's form

(iii) Solution of the PDE (D + 2D′ – 3) z = 0 is :

(a) $z = e^{-3x}F(y + 2x)$

(b) $z = e^{3x}F(y - 2x)$

(c) $z = e^{3x}F(2y - x)$

(d) $z = e^{-3x}F(2y - x)$

(iv) Particular integral of the PDE $(D^2 - D')z = e^{x-2y}$ is :

(a) $\frac{1}{3}e^{x-2y}$

(b) e^{x-2y}

(c) $-\frac{1}{3}e^{x-2y}$

(d) 0

(v) If $S^2 - 4RT < 0$, then reduced canonical form of the PDE Rr + Ss + Tt + F(x, y, z, p, q) = 0 is :

(a) Parabolic

(b) Hyperbolic

(c) Elliptic

(d) None of these

(vi) A PDE $Z_{xx} + Z_{xy} - Y^2Z_x = e^{xy^2}$ is :

(a) Hyperbolic (b) Elliptic

(c) Parabolic (d) None of these

(vii) The maximum point and the minimum point of a function f(x) are called the :

(a) Stationary points (b) Critical points

(c) Extremum points (d) None of these

(viii) A function for which $\delta_I = 0$ are called :

(a) Continuous function (b) Identity function

(c) Linear function (d) Stationary functions

(ix) PDE $c^2u_{xx} = u_{tt}$ is :

(a) Wave equation (b) Heat equation

(c) Laplace equation (d) None of these

(x) The general form of the first order PDE is :

(a) $f(x, y, p, q) = 0$ (b) $F(x, p) = G(y, q)$

(c) $f(z, p, q) = 0$ (d) $f(x, y, z, p, q) = 0$ 10

UNIT—I

2. (a) Solve the PDE $x(y^2 - z^2)p + y(z^2 - x^2)q = z(x^2 - y^2)$. 6

(b) Obtain the PDE from equation $f(x + y + z, x^2 + y^2 + z^2) = 0$. 4

3. (p) Solve the PDE $z^2(p^2 + q^2) = x^2 + y^2$. 5

(q) Solve the PDE $z^2 = pqxy$ by Charpit's method. 5

UNIT—II

4. (a) Solve the PDE $r + s - 6t = y \cos x$. 5

(b) Solve the PDE $(D^2 - 4D'^2)z = \frac{4x}{y^2} - \frac{y}{x^2}$. 5

5. (p) Solve the PDE $D(D - 2D' - 3)z = e^{x+2y}$. 5

(q) Solve the PDE $x^2\frac{\partial^2 z}{\partial x^2} - y^2\frac{\partial^2 z}{\partial y^2} = x^2y$. 5

UNIT—III

6. (a) Solve the PDE $r - a^2t = 0$ by the Monge's method. 5

(b) Reduce the Tricomi equation $Z_{xx} + xZ_{yy} = 0, x \neq 0$ to canonical form. 5

7. (p) Reduce the equation $y^2r - 2xys + x^2t = \frac{y^2}{x}p + \frac{x^2}{y}q$ to canonical form and hence solve it. 5

(q) Solve the PDE $r + t - rt + s^2 = 1$ by Monge's method. 5

UNIT—IV

8. (a) Define the terms :

(i) Curves close in the sense of proximity of the zeroth order

(ii) Curves close in the sense of the first order proximity. 2+2=4

(b) State and prove the necessary condition for an extremum of a functional. 2+4=6

9. (p) Define n^{th} order distance. Find the distance between the curve $y(x) = xe^{-x}$, $y_1(x) = 0$ on $[0, 2]$. 1+4=5

(q) Define k^{th} order proximity. Show that the curves $y(x) = \frac{\sin x}{n}$, where n is sufficiently large, and $y_1(x) = 0$ on $[0, \pi]$ are closed in the sense of proximity of any order. 1+4=5

UNIT—V

10. (a) By the method of separation of variables solve the equation $\frac{\partial u}{\partial x} + 2\frac{\partial u}{\partial t} + u$. Given that $u(x, 0) = 6e^{-3x}$. 5

(b) Solve the boundary value problem $c^2u_{xx} = u_t$ for negative constant (i.e. $\lambda = -k^2$). Given that :

(i) $u(0, t) = 0, u(\ell, t) = 0$, for all t

(ii) $u(x, 0) = f(x) = \begin{cases} x, & 0 < x < \ell/2 \\ \ell - x, & \ell/2 < x < \ell \end{cases}$. 5

11. (p) Solve by separation method :

$$\frac{\partial^2 z}{\partial x^2} - 2\frac{\partial z}{\partial x} + \frac{\partial z}{\partial y} = 0.$$ 5

(q) A tightly stretched string with fixed end points $x = 0$ and $x = \ell$ in the shape defined by $y = kx(\ell - x)$ where k is constant, released from this position of rest. Find y(x, t), if the vertical displacement is $\frac{\partial^2 y}{\partial t^2} = c^2\frac{\partial^2 y}{\partial x^2}$. 5