Third Semester B. Sc. II Examination

(Old Course)

MATHEMATICS

Paper – VI

(Partial Differential Equations)

P. Pages : 7

Time : Three Hours] [Max. Marks : 60

Note : (1) Question no.**One** is compulsory.
(2) Solve **One** question from each unit.

1. Choose the correct alternative :—

(i) An equation of the form $Pp+Qq=R$ (where P, Q, R are functions of x, y, z, is called

(a) Charpit's auxiliary equation

(b) Lagrange's linear equation

(c) Jacobi's auxiliary equation

(d) None of these.

(ii) The solution of $r = a^2t$ is ———

(a) $Z=F_1(y+ax)+F_2(y-ax)$

(b) $Z=F(y-ax)$

(c) $Z = F(y+ax)$

(d) $Z = F_1(y+ax) + xF_2(y-ax)$.

(iii) The particular integral of the PDE $(D-D'-1)(D-D'-2)Z = e^{2x-y}$ is ———

(a) $\frac{e^{2x-y}}{2}$ (b) $\frac{e^{2x-y}}{4}$

(c) e^{2x-y} (d) $-\frac{e^{2x-y}}{2}$

(iv) A point $x = x_0$ for which $f'(x_0) = 0$ is called

(a) A point of inflexion

(b) Stationary point

(c) Extremum point

(d) None of these

(v) The PDE $\nabla^2 u = 0$ is

(a) Heat equation

(b) Wave equation

(c) Laplace equation

(d) None of these.

(vi) An extremum of a functional $I[y(x)]$ on the entire set M is called.

(a) Weak relative extremum

(b) Absolute extremum

(c) Strong relation extremum

(d) None of these.

(vii) The PDE of the form $Rr+Ss+Tt+f(x, y, z, p, q) = 0$ is elliptic at the point (x, y) if ——

(a) $S^2-4RT>0$ (b) $S^2-4RT<0$

(c) $S^2-4RT=0$ (d) None of these

(viii) The solution of $S = 0$ is ——

(a) $f(x)+g(y)$ (b) $f(x)+g(x)$

(c) $xf(y)+g(x)$ (d) $yf(x)+g(x)$

(ix) Which of the following PDE is non homogeneous :—

(a) $(D+D'-1)z=e^{2x-y}$

(b) $(xD+yD')z=\text{Sin}(x-y)$

(c) $(D^2-2DD'+yD'^2)z=\ln(x^2+y)$

(d) $(y^2D^3-4D^2D' - 3DD'^2)z=\tan x^2$.

(x) If the characteristic equation of the PDE of the form $Rr+Ss+Tt+f(x, y, z, p, q)=0$ has distinct real roots, then the PDE is ——

(a) Elliptic (b) Hyperbolic

(c) Parabolic (d) None of these.

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UNIT I

2. (a) Form the PDE by eliminating the arbitrary function from the equation.
$f(x+y+z, x^2+y^2+z^2)=0$ 5

(b) Apply Charpit's method to find the complete solution of
$(p^2+q^2)y=qz$ 5

3. (p) Find the general integral of
$(mz-ny)P+(nx-lz)q+mx-ly=0$ 5

(q) Use Jacobi's method to solve the PDE
$xpq+yq^2=1$ 5

UNIT II

4. (a) Solve : $(D^2+DD'+D'^2)z = x^2+xy+y^2+e^{x+y}$ 5

(b) Solve : $(D^2-DD')z = \cos x . \cos 2y$ 5

5. (p) Solve : $r+s-6t=y \cos x$ 5

(q) Solve : $(2D^2-DD'-3D'^2)z=5e^{x-y}$ 5

UNIT III

6. (a) Solve $r-a^2t=0$ by Monge's method. 5

(b) Show that the equation $z_{xx}+xz_{yy}=0$, $x \neq 0$ is hyperbolic for $x<0$ and reduce it to canonical form. 5

7. (p) Solve $r - t\cos^2 x + p\tan x = 0$ by Monge's method. 5

(q) Reduce the equation

$$y^2r-2xys+x^2t=\frac{y^2}{x}p+\frac{x^2}{y}q$$ to canonical form 5

UNIT IV

8. (a) Find the distance between the curves $y(x)=xe^{-x}$, $y_1(x)=0$ on $[0, 2]$ 5

(b) Show that the functional

$$I[y(x)]=\int_0^1 \{2y(x)+y'(x)\}\,dx$$ defined in the space $C_1[0, 1]$ is continuous on the function $y_0(x)=x$ in the sense of first order proximity. 5

9. (p) Define linear functional and show that the functional $L[y(x)] = \int_a^b \{2y(x) - 3y'(x)\}dx$ defined on M is linear. 5

(q) Show that the functional

$I[y(x)] = \int_0^1 (x^2 - y^2)\, dx$ attains a strict maxima on the curve $y(x) = 0$ 5

UNIT V

10. (a) Solve : $\frac{\partial^2 z}{\partial x^2} + z = 0$ by the method of separation of variables. Given that $x = 0$ $z = e^y$ and $\frac{\partial z}{\partial x} = 1$ 5

(b) Solve by separation method.

$$x \frac{\partial^2 u}{\partial x . \partial y} + 2yu = 0$$ 5

11. (p) Solve $\frac{\partial^2 z}{\partial x . \partial y} = \sin x . \sin y$ for which $\frac{\partial z}{\partial y} = -2 \sin y$ when x=0 and z=0, when y is an odd multiple of $\pi/2$. 5

(q) A tightly stretched string with fixed end points $x=0$ and $x=l$ in the shape defined by $y=Kx\ (l-x)$, where K is constant, released from this position of rest.
Find $y(x, t)$, if the vertical displacement is $\frac{\partial^2 y}{\partial t^2} = C^2 \frac{\partial^2 y}{\partial x^2}$ 5