AS–1399

B.Sc. Part–II (Semester–III) Examination

MATHEMATICS (New)

(Advanced Calculus)

Paper—V

Time : Three Hours] [Maximum Marks : 60

Note :—(1) Question No. **1** is compulsory, attempt once.

(2) Attempt **ONE** question from each unit.

1. Choose the correct alternative :

(i) If the limit of a sequence exists, the sequence is said to be ________.

(a) Unbounded (b) Convergent

(c) Divergent (d) Oscillatory 1

(ii) The sequence defined by $s_n = \frac{1}{n+1}$ is bounded and ______.

(a) Monotone increasing (b) Monotone decreasing

(c) Oscillatory (d) None of these 1

(iii) Let $\sum a_n$ be a series of positive terms such that $\lim_{n \to \infty} \sqrt[n]{a_n} = \ell$, $a_n \geq 0, \forall$ n. Then $\sum a_n$ is convergent if :

(a) $\ell = 1$ (b) $\ell < 1$

(c) $\ell > 1$ (d) $\ell = 0$ 1

(iv) The series $x_n = \frac{1}{n^2+2}$ is :

(a) Convergent (b) Divergent

(c) Oscillatory (d) None of these 1

(v) If $\lim_{(x, y) \to (x_0, y_0)} f(x, y) \neq f(x_0, y_0)$ then :

(a) f is continuous

(b) f is discontinuous

(c) function f fails to be continuous at (x_0, y_0)

(d) Both (b) and (c) 1

(vi) The neighbourhood $N_\delta(x_0, y_0) - \{(x_0, y_0)\}$ is called as :

(a) δ-nbd

(b) Rectangular nbd of (x_0, y_0)

(c) Deleted δ-nbd

(d) None of these 1

(vii) If $x = r \cos \theta$ $y = r \sin \theta$ then Jacobian $J = \frac{\partial(x, y)}{\partial(r, \theta)}$ is :

(a) r

(b) $\frac{1}{r}$

(c) r^2

(d) $\frac{1}{r^2}$ 1

(viii) Let (x_0, y_0) be a critical point of a function of two variables which is defined in the open region $D \subseteq R^2$ and have continuous second order partial derivative in D. Then $rt - s^2 = 0 \Rightarrow$ ______.

(a) f has local maximum at (x_0, y_0)

(b) f has local minimum at (x_0, y_0)

(c) f has neither maximum nor minimum at (x_0, y_0)

(d) the test is inconclusive 1

(ix) In transforming double integral to polar co-ordinates we use dxdy =

(a) $dr d\theta$

(b) $r dr d\theta$

(c) $\frac{1}{r} dr d\theta$

(d) $\frac{dr}{d\theta}$ 1

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(x) The value of $\int_0^1\int_0^1\int_0^1 dxdydz$ is :

(a) 1 (b) 0

(c) 2 (d) 3 1

UNIT—I

2. (a) Every convergent sequence of real numbers is a Cauchy Sequence. Prove this. 3

(b) Let $<s_n>$ be a sequence such that $\lim_{n \to \infty} s_n = \ell$ and $s_n \geq 0 \ \forall \ n \in N$. Then prove $\ell \geq 0$. 3

(c) Show that the sequence $<s_n>$ defined by $s_n = \frac{1}{n+1} + \frac{1}{n+2} + ... + \frac{1}{n+n}$ converges. 4

3. (p) Prove that a monotonic sequence of real numbers is convergent if and only if it is bounded. 4

(q) Evaluate $\lim_{n \to \infty} s_n$ for $s_n = \sqrt{n+a} - \sqrt{n+b}, a \neq b$. 3

(r) Let $<x_n>$ be a sequence of real numbers and for each $n \in N$. Let $s_n = x_1 + x_2 + ... + x_n$ and $t_n = |x_1| + |x_2| + ... + |x_n|$. Prove that if $<t_n>$ is a Cauchy sequence then $<s_n>$ is Cauchy sequence. 3

UNIT—II

4. (a) Show that $\sum \frac{1}{(2n+1)^3}$ is convergent and $\sum \frac{1}{(2n-1)^{1/2}}$ is divergent. 4

(b) Let $\sum_{n=1}^{\infty} a_n$ be a sequence of real numbers such that $\ell = \lim_{n \to \infty} \sqrt[n]{a_n}$, $a_n \geq 0, \forall \ n$. Then prove that $\sum a_n$ is convergent if $\ell < 1$. 4

(c) A series $\sum x_n$ of non-negative terms then prove that the sequence $<s_n>$ of partial sum is monotonic increasing. 2

5\. (p) Show that an absolutely convergent series is convergent but its converse necessarily does not hold. 4

(q) Test the convergence of the series $\sum_{n=2}^{\infty} \frac{1}{n(\log n)^p}$, $p > 0$ by Cauchy's Integral Test. 4

(r) Test the convergence of $\frac{1}{1.2} + \frac{1}{2.3} + \frac{1}{3.4} + \ldots$ 2

UNIT—III

6\. (a) Let f(x, y) be defined and continuous in the open region D and let $f(x_1, y_1) = z_1$, $f(x_2, y_2) = z_2$, $z_1 \neq z_2$. Then for every number z_0 between z_1 and z_2, there is a point (x_0, y_0) of D for which $f(x_0, y_0) = z_0$, prove this. 4

(b) Using $\epsilon - \delta$ definition of a limit of a function, prove that $\lim_{(x, y) \to (4, -1)} (3x - 2y) = 14$. 3

(c) Expand $f(x, y) = x^2 - y^2 + 3xy$ at the point (1, 2) by using Taylor's theorem. 3

7\. (p) Let real valued functions f and g be continuous in an open set $D \subseteq R^2$. Then prove that f + g is continuous in D. 3

(q) Let $f(x, y) = \frac{x^2y^2}{x^2y^2 + (x - y)^2}$, $x^2y^2 + (x - y)^2 \neq 0$. Show that limit of the function f as $(x, y) \to (0, 0)$ does not exist even though iterated limits are equal. 4

(r) Expand e^{xy} at the point (2, 1) up to first three terms. 3

UNIT—IV

8\. (a) If xu = yz, yv = xz, zw = xy, find $\frac{\partial(x, y, z)}{\partial(u, v, w)}$. 3

(b) Find the least distance of the origin from the plane x – 2y + 2z = 9 by using Lagrange's method of multipliers. 4

(c) Find the extremum of sin A sin B sin C subject to the condition A + B + C = π. 3

9. (p) Let f(x, y) be defined in an open region D and it has a local maximum or local minimum at (x_0, y_0); if the partial derivative f_x and f_y exist at (x_0, y_0), then $f_x(x_0, y_0) = 0$ and $f_y(x_0, y_0) = 0$. Prove this. 3

(q) If $x + y = 2e^{\theta} \cos \phi$, $x - y = 2ie^{\theta} \sin \phi$, show that $JJ' = 1$. 3

(r) Use the method of Lagrange multiplier to locate all local maxima and minima and also find the absolute maximum or minimum of $f(x, y) = x^2 + y^2$, where $x^4 + y^4 = 1$. 4

UNIT—V

10. (a) Evaluate $\iint_S \overline{F}\cdot\overline{n}\,ds$ where $\overline{F} = (x^2 - yz)i + (y^2 - zx)j + (z^2 - xy)k$ and S is surface of rectangular parallelopiped $0 \le x \le a$, $0 \le y \le b$, $0 \le z \le c$ by Gauss-divergence theorem. 5

(b) Apply Stoke's theorem to evaluate $\oint_C [(x + y)dx + (2x - z)dy + (y + z)dz]$, where C is the boundary of the triangle with vertices (2, 0, 0), (0, 3, 0), (0, 0, 6). 5

11. (p) Evaluate the Double integral $\int_0^{\log 8}\int_0^{\log y} e^{x+y}\,dxdy$. 3

(q) Change the order of $\iint_D f(x, y)\,dxdy$, where D is bounded by parabolas $y^2 = x$ and $x^2 = y$. 3

(r) Evaluate $\int_0^1 \int_0^{2(1-x)} \int_0^{2(1-x)-y} x^2 y\,dzdydx$. 4

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