AT–325

B.Sc. (Part-II) Semester-III Examination
MATHEMATICS (New)
(Elementary Number Theory)
Paper—VI

Time : Three Hours] [Maximum Marks : 60

Note :— (1) Question No. **1** is compulsory. Attempt it at once only.

(2) Attempt **ONE** question from each unit.

1. Choose the correct alternative (**1** mark each) : 10

(1) If $c > 0$ is common divisor of a and b, then $\left(\frac{a}{c}, \frac{b}{c}\right)$ is equal to :

(a) $\frac{(a, b)}{c}$ (b) $\frac{[a, b]}{c}$

(c) $\frac{c}{(a, b)}$ (d) $\frac{c}{[a, b]}$

(2) The product of any m consecutive integers is divisible by :

(a) $(m + 1)\,!$ (b) $(m - 1)\,!$

(c) $m\,!$ (d) $\left(\frac{m}{2}\right)!$

(3) If $x > 0$, $y > 0$ and $x - y$ is an even, then $(x^2 - y^2)$ is divisible by :

(a) 3 (b) 4

(c) 5 (d) 7

(4) If $n > 2$ is a positive integer, then $1^3 + 2^3 + ... + (n - 1)^3 \equiv$

(a) 0 (mod n) (b) 1 (mod n)

(c) 2 (mod n) (d) None of these

(5) If (a, b) = 1 then integers a and b are :

(a) Prime
(b) Relatively Primes
(c) Compositive
(d) None of these

(6) An integer 'r' is root of f(x) modulo p if :

(a) $f(r) \equiv 1 \pmod p$
(b) $f(r) \equiv 0 \pmod p$
(c) $f(r) \equiv 2 \pmod p$
(d) $f(r) \equiv p \pmod 2$

(7) The number of quadratic non residues modulo 23 is :

(a) 10
(b) 22
(c) 11
(d) 2

(8) The congruence $x^n \equiv 2 \pmod{13}$ has a solution for x if :

(a) n = 5
(b) n = 7
(c) n = 6
(d) n = 8

(9) If p is a quadratic residue of an odd prime q, then q is a :

(a) quadratic residue of p
(b) quadratic residue of q
(c) prime
(d) residue of p

(10) By Fermat's theorem when 8^{103} is divided by 103, the remainder is :

(a) 103
(b) 8
(c) 9
(d) 10

UNIT—I

2. (a) If x and y are odd, prove that $x^2 + y^2$ is not a perfect square. 4

(b) Prove that, if $c \mid a$ and $c \mid b$, then $c \mid (a, b)$. 3

(c) Find the values of x and y to satisfy the equation $423x + 198y = 9$. 3

3. (p) If (a, b) = 1, then prove that (ac, b) = (c, b). 4

(q) For positive integers a and b, prove that :

(a, b) [a, b] = ab. 3

(r) Find :

(5325, 492). 3

UNIT—II

4. (a) Prove that every positive integer greater than one has at least one prime divisor. 4

(b) Prove that :

$(a^2, b^2) = c^2$ if $(a, b) = c$. 3

(c) If P_n is the n^{th} prime number then show that :

$P_n \leq 2^{2^{n-1}}$. 3

5. (p) If m and n are distinct non-negative integers, then prove that $(F_m, F_n) = 1$. 5

(q) Find the solution of the linear Diophantine equation :

$10x + 6y = 110$. 5

UNIT—III

6. (a) Prove that congruence is an equivalence relation. 5

(b) Show that 41 divides $2^{20} - 1$. 5

7. (p) Solve the system of three congruences $x \equiv 2 \pmod 3$, $x \equiv 3 \pmod 5$ and $x \equiv 2 \pmod 7$. 5

(q) If f is a polynomial with integral coefficients and $a \equiv b \pmod m$, then prove that :

$f(a) \equiv f(b) \pmod m$. 5

UNIT—IV

8. (a) If p is a prime and k is a positive integer, then prove that $\phi(p^k) = p^k\left(1 - \frac{1}{p}\right)$. 5

(b) If m is a positive integer and a is an integer with $(a, m) = 1$, then prove that :

$a^{\phi(m)} \equiv 1 \pmod m$. 5

9. (p) Prove that Möbius μ-function is multiplicative. 4

(q) Find the value of $\phi(300)$. 3

(r) Find the value of $\tau(1800)$ and $\sigma(1800)$. 3

UNIT—V

10. (a) If $(a, m) = d > 1$, then prove that m has no primitive root of a. 5

(b) Prove that if r is a quadratic residue modulo $m > 2$, then $r^{\phi(m)/2} \equiv 1 \pmod m$. 5

11. (p) Let a be an odd integer, then prove that $x^2 \equiv a \pmod 4$ has a solution if and only if $a \equiv 1 \pmod 4$. 5

(q) If $m > 2$ and $n > 2$ are the integers with $(m, n) = 1$, then prove that mn has no primitive roots. 5