AT–324

B.Sc. (Part-II) Semester-III Examination

MATHEMATICS (New)

(Advanced Calculus)

Paper—V

Time : Three Hours] [Maximum Marks : 60

Note :—(1) Question No. **1** is compulsory, attempt once.

(2) Attempt **ONE** question from each unit.

1. Choose the correct alternative :

(1) The sequence $< s_n >$; where $s_n = r^n$ converges to zero if : 1

(a) $| r | < 1$ (b) $| r | > 1$

(c) $| r | = 1$ (d) None of these

(2) The value of $\lim_{n\to\infty} \frac{3^n}{2^{2n}}$ is : 1

(a) 2 (b) 1

(c) 0 (d) 4

(3) Let Σa_n be a series of positive terms such that $\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = \ell \ \forall_n$; then the series Σa_n is convergent if : 1

(a) $l = 1$ (b) $l < 1$

(c) $l > 1$ (d) None of these

(4) The series $1+\frac{1}{2}+\frac{1}{3}+\ldots\ldots+\frac{1}{n}+\ldots\ldots$ is called : 1

(a) Geometric series (b) Harmonic series

(c) Arithmetic series (d) None of these

(5) The value of $\lim_{x \to 2} \left\{ \lim_{y \to 1} (xy - 3x + 4) \right\}$ is : 1

(a) 4
(b) 3
(c) 1
(d) 0

(6) The value of δ in the following expression $0 < |(x, y) - (0, 0)| < \delta \Rightarrow |x^2 + y^2| < \frac{1}{100}$ is : 1

(a) $\frac{1}{100}$
(b) $\frac{1}{10}$
(c) 1
(d) None of these

(7) A function f(p) is said to have absolute maximum at $P_0 \in D$ iff for all $P \in D$ satisfies the condition : 1

(a) $f(P_0) \leq f(P)$
(b) $f(P_0) = f(P)$
(c) $f(P_0) \geq f(P)$
(d) None of these

(8) If $x = r \cos \theta$ and $y = r \sin \theta$ then $\frac{\partial(x, y)}{\partial(r, \theta)}$ is : 1

(a) r
(b) $\frac{1}{r}$
(c) r^2
(d) $\frac{1}{r^2}$

(9) The value of $\int_0^1 \int_0^2 \int_0^3 dxdydz$ is : 1

(a) 6.
(b) 2
(c) 1
(d) 3

(10) If $\bar{F} = y\bar{i} + x\bar{j} + z^2\bar{k}$ then div $\bar{F}$ at (1, 1, 1) is : 1

(a) 2
(b) 1
(c) 0
(d) 3

UNIT—I

2. (a) If $\lim_{n\to\infty} s_n = \ell$ and $\lim_{n\to\infty} t_n = m$ then prove that :

$$\lim_{n\to\infty} s_n t_n = \ell m.$$ 4

(b) Let $< s_n >$ be a sequence such that $\lim_{n\to\infty} s_n = \ell$ and $s_n \geq 0$, then prove that $l \geq 0$. 3

(c) Prove that :

$$\lim_{n\to\infty} \frac{1+2+3+........+n}{n^2} = \frac{1}{2}.$$ 3

3. (p) Prove that limit of sequence if it exist is unique. 4

(q) Prove that the sequence $< s_n >$, $s_n = \frac{1}{1!} + \frac{1}{2!} + \frac{1}{3!} + ... + \frac{1}{n!}$ is monotonic and bounded. 3

(r) Show that the sequence $< s_n >$ defined by $s_n = 1 + \frac{1}{2} + \frac{1}{3} + ... + \frac{1}{n}$ does not converge. 3

UNIT—II

4. (a) Prove that the Geometric series $\sum_{n=1}^{\infty} ar^{n-1}$ is converges to $\frac{a}{1-r}$ if $0 < r < 1$ and diverges for $r \geq 1$. 4

(b) Test the converges of the series $\sum_{n=1}^{\infty} \frac{n}{2n^3 - 1}$. 3

(c) Discuss the convergence of the series $\sum_{n=1}^{\infty} \frac{n!}{n^n}$. 3

5. (p) Let Σa_n be a series of positive terms such that $\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = \ell$. Then show that the series Σa_n is convergent if $l < 1$ and diverges when $l > 1$. 4

(q) Test the convergence of the series $\sum\left(\frac{n}{n+1}\right)^{n^2}$. 4

(r) Discuss the convergence of the series $\sum \frac{1}{4^n+1}$. 2

UNIT—III

6. (a) Prove that if limit of a function f(x, y) as $(x, y) \to (x_0, y_0)$ exist then it is unique. 4

(b) Using ϵ-δ definition, prove that :

$$\lim_{(x,y)\to(1,1)} (x^2+2y)=3.$$ 3

(c) Expand $x^3 + y^3 - 3xy$ in powers of $(x - 2)$ and $(y - 3)$. 3

7. (p) Using ϵ–δ definition of continuity, prove that f(x, y) = x + y is continuous for all (x, y) in xy-plane. 4

(q) Prove that $\lim_{(x,y)\to(4,-1)} (3x-2y)=14$; by using $\epsilon - \delta$ definition. 3

(r) Expand e^{xy} at the point (2, 1) upto first three terms. 3

UNIT—IV

8. (a) A rectangular box open at the top is to have a volume of 32 cubic feet. What must be the dimensions of the box if the surface area is minimum ? 4

(b) Find the extreme values of $x^3 + y^3 - 3dxy$. 3

(c) If $u = \frac{x+y}{1-xy}$ and $v = \tan^{-1}x + \tan^{-1}y$, find $\frac{\partial(u, v)}{\partial(x, y)}$; if $xy \neq 1$. State whether u and v are functionally related. If so, find the relationship. 3

9. (p) Find the coordinates of the foot of the perpendicular drawn from the point P(6, 2, 3) to the plane z = 5x − y + 2; by minimizing the square of the distance from P to any point (x, y, z) in the plane. 4

(q) Let the function f(x, y) be defined and continuous on an open region D of xy-plane. If f(x, y) has local maximum or minimum at $P_0(x_0, y_0)$ in D and f(x, y) is differentiable at P_0 then prove that $\frac{\partial f}{\partial x}=\frac{\partial f}{\partial y}=0$ at $P_0(x_0, y_0)$. 4

(r) If $x = r\cos\theta$, $y = r\sin\theta$ then find :

$$\frac{\partial(x, y)}{\partial(r, \theta)}.$$ 2

UNIT—V

10. (a) Evaluate $\int_0^1 \int_{x^2}^{2-x} xy\,dy\,dx$; by changing the order of integration. 5

(b) Evaluate $\iiint_R x^2\,dx\,dy\,dz$, where R is a cube bounded by the planes $z = 0$, $z = a$, $y = 0$, $y = a$, $x = 0$, $x = a$. 5

11. (p) Verify Gauss divergence theorem for the function $\overline{F} = y\overline{i} + x\overline{j} + z^2\overline{k}$; over the region bounded by $x^2 + y^2 = 4$; $z = 0$ and $z = 2$. 5

(q) Verify Stoke's Theorem for the function $\overline{F} = x^2\overline{i} + xy\overline{j}$ integrated round the square in the plane $z = 0$ and bounded by the lines $x = 0$, $y = 0$, $x = 2$, $y = 2$. 5