AU–81

B.Sc. Part—II (Semester—III) Examination

MATHEMATICS (New)

Paper—V

(Advanced Calculus)

Time : Three Hours] [Maximum Marks : 60

Note :— (1) Question No. 1 is compulsory. Attempt once.

(2) Attempt ***one*** question from each unit.

1. Choose the correct alternative :

(i) A sequence $<S_n>$ is strictly increasing if _____ $\forall$ n $\in$ N.

(a) $S_n = S_{n+1}$ (b) $S_n \leq S_{n-1}$

(c) $S_n < S_{n+1}$ (d) $S_n > S_{n+1}$ 1

(ii) Let $\{x_n\}$ be a Cauchy sequence of real numbers. Then the sequence $\{\cos x_n\}$ is ____.

(a) Unbounded (b) Bounded but not Cauchy

(c) Cauchy but not bounded (d) Cauchy sequence 1

(iii) The P-series $\sum \frac{1}{n^P}$ is convergent for ____.

(a) $P < 1$ (b) $P > 1$

(c) $P = 1$ (d) $P = 0$ 1

(iv) The series $\sum x_n = \sum \frac{1}{4^n + 1}$ is ______.

(a) Convergent (b) Divergent

(c) Harmonic (d) None of these 1

(v) If iterated limits of a function are not equal at point then :

(a) Limit exist at that point
(b) Limit does not exist
(c) Limit is zero
(d) None of these 1

(vi) If $\lim_{P \to P_0} f(P) = f(P_0)$ then :

(a) f is continuous at P_0
(b) f is discontinuous at P_0
(c) f is continuous at P
(d) None of these 1

(vii) If $u = 2x - y$, $v = x + 4y$ then $J = \dfrac{\partial(u, v)}{\partial(x, y)} =$ ______

(a) $\dfrac{1}{9}$
(b) -9
(c) 9
(d) 9^2 1

(viii) The function f(x, y) has an absolute maxima at a point (x_0, y_0) in D if ____ for all $(x, y) \in D$.

(a) $f(x, y) \leq f(x_0, y_0)$
(b) $f(x, y) \geq f(x_0, y_0)$
(c) $f_x(x, y) \leq f_y(x_0, y_0)$
(d) None of these 1

(ix) The series $\sum a r^{n-1}$ is convergent if :

(a) $r = 1$
(b) $r < 1$
(c) $r > 1$
(d) None of these 1

(x) $\int_1^2 \int_1^3 xy^2 \, dxdy =$ ______ .

(a) $\dfrac{24}{3}$
(b) $\dfrac{26}{3}$
(c) $\dfrac{28}{3}$
(d) 10 1

UNIT—I

2. (a) Let $<x_n>$ be a sequence of real numbers that converges to $x \neq 0$. Then prove that $\lim_{n \to \infty} \left(\frac{1}{x_n}\right) = \frac{1}{x}$, for $x_n \neq 0 \ \forall \ n \in N$. 4

(b) Show that the sequence $<S_n>$ defined by $S_n = \frac{1}{3+1} + \frac{1}{3^2+1} + ... + \frac{1}{3^n+1}$ is monotonic and bounded. 3

(c) Every convergent sequence of real numbers is a Cauchy sequence. Prove this. 3

OR

3. (p) Show that the sequence $<S_n>$ defined by $S_n = 1 + \frac{1}{2} + \frac{1}{3} + ... + \frac{1}{n}$ does not converge. 3

(q) Let $<S_n>$ be a sequence such that $\lim S_n = \ell$ and $S_n > 0 \ \forall \ n \in N$. Then $\ell \geq 0$. Prove this. 3

(r) A real sequence $<S_n>$ converges if and only if for each $\in > 0$, $\exists \ M \in N$ such that $|S_m - S_n| < \in \ \forall \ m, n \geq M$. Prove this. 4

UNIT—II

4. (a) Let $\sum x_n$ be a positive term series such that $\lim_{n \to \infty} \frac{x_{n+1}}{x_n} = \ell$.

Then the series converges if $\ell < 1$. Prove this. 4

(b) Test the convergence :

$$\frac{1}{1.2} + \frac{1}{2.3} + \frac{1}{3.4} + ...$$ 3

(c) Define :

(i) Absolutely convergent

(ii) Harmonic series

(iii) Conditionally convergent. 3

OR

5. (p) If $<a_n>$ with $a_n \geq 0, n \in N$ is monotonic decreasing sequence and if $\sum_{n=1}^{\infty} b_n$ is convergent then the series $\sum_{n=1}^{\infty} a_n b_n$ is also convergent. Prove this. 4

(q) Test the convergence by integral test :

$$\sum_{n=1}^{\infty} \frac{1}{n^2+2}.$$ 3

(r) Let $\sum_{n=1}^{\infty} a_n$ be a sequence of real numbers such that $\ell = \lim_{n\to\infty} \sqrt[n]{a_n}$, $a_n \geq 0 \ \forall$ n. Then $\sum_{n=1}^{\infty} a_n$ diverges if $\ell > 1$. Prove this. 3

UNIT—III

6. (a) Let $f(x, y) = \frac{x^3}{x^2+y^2}$, for $(x, y) \neq (0, 0)$

$= 0$, for $(x, y) = (0, 0)$

Using $(\in, \delta)$ definition, prove that f is continuous at (0, 0). 3

(b) Expand $x^3 + y^3 - 3xy$ in power of $x - 2$ and $y - 3$ i.e. at the point (2, 3). 4

(c) If limit of a function f(x,y) as $(x, y) \to (x_0, y_0)$ exists, then it is unique. Prove this. 3

OR

7. (p) Let real valued functions f and g be continuous in an open set $D \subset R^2$. Then prove that $f - g$ is continuous in D. 4

(q) Expand $f(x, y) = x^3 + y^3 + 3xy$ in power of $x - 1$ and $y - 1$. 3

(r) Using $\in - \delta$ definition of a limit of a function, prove that $\lim_{(x,y)\to(1,1)}(x^2+2y) = 3$. 3

UNIT—IV

8. (a) Show that the function $f(x, y) = 2x^4 - 3x^2y + y^2$ has neither maxima nor minima at (0, 0). 3

(b) If $x + y + z = u$, $y + z = uv$, $z = uvw$, prove that $\frac{\partial(x,y,z)}{\partial(u,v,w)} = u^2v$. 3

(c) Find by using Lagrange's method of multipliers, the least distance of the origin from the plane $x - 2y + 2z = 9$. 4

OR

9. (p) If x, y are differentiable functions of u, v and u, v are differentiable functions of r and s then prove that : $\frac{\partial(x,y)}{\partial(r,s)} = \frac{\partial(x,y)}{\partial(u,v)} \cdot \frac{\partial(u,v)}{\partial(r,s)}$. 5

(q) Let f(x, y) be defined in an open region D and it has local maximum or local minimum at (x_0, y_0). If the partial derivatives f_x and f_y exist at (x_0, y_0), then $f_x(x_0, y_0) = 0$ and $f_y(x_0, y_0) = 0$, prove this. 5

UNIT—V

10. (a) Evaluate by changing the order of integration :

$$\int_0^1 \int_x^{\sqrt{2-x^2}} \frac{x\,dydx}{\sqrt{x^2+y^2}}.$$ 5

(b) Evaluate $\int_0^{2a} \int_0^{\sqrt{2ax-x^2}} (x^2+y^2)dy\,dx$. 5

OR

11. (p) Evaluate $\iint_S \bar{F}.\bar{n}\,ds$ where $\bar{F} = axi + byj + czk$ and S is the surface of sphere $x^2+y^2+z^2=1$. 5

(q) Evaluate by Stokes theorem $\int_C e^x dx + 2ydy - dz$, where C is the curve $x^2+y^2 = 4$, $z = 2$. 5