AS–1398

B.Sc. Part—II (Semester—III) Examination

MATHEMATICS (Old) (Upto S/17)

(Advanced Calculus)

Paper—V

Time : Three Hours] [Maximum Marks : 60

Note :—(1) Question No. **1** is compulsory.

(2) Attempt **ONE** question from each Unit.

1. Choose correct alternatives :

(i) The sequence $<s_n>$, where $s_n = \frac{\sqrt{n}}{n+1}$ is :

(a) Monotonic decreasing (b) Monotonic increasing

(c) Constant sequence (d) Oscillatory sequence 1

(ii) If $<s_n>$, $<t_n>$ and $<u_n>$ be three sequences such that $s_n \le t_n \le u_n\ \forall\ n \in N$ and $\lim_{n\to\infty} s_n = \lim_{n\to\infty} u_n = \ell$ then $\lim_{n\to\infty} t_n$ is :

(a) 0 (b) ℓ

(c) $-\ell$ (d) 1 1

(iii) The geometric series $\sum_{n=1}^{\infty} x^{n-1}$ is convergent if :

(a) $x = 0$ (b) $x = 1$

(c) $x < 1$ (d) $x > 1$ 1

(iv) The series $\sum \frac{1}{n^p}$ is convergent if :

(a) $p \le 1$ (b) $p > 1$

(c) $p = 0$ (d) $p = -1$ 1

(v) If $x = r \cos \theta$, $y = r \sin \theta$ then the value of $\frac{\partial(x, y)}{\partial(r, \theta)}$ is :

(a) r (b) $-r$

(c) $r^2\theta$ (d) $r\theta$ 1

(vi) The limits $\lim_{x \to x_0}\left[\lim_{y \to y_0} F(x, y)\right]$ and $\lim_{y \to y_0}\left[\lim_{x \to x_0} F(x, y)\right]$ are called as :

(a) Left hand and right hand limits (b) Double limits

(c) Repeated or iterated limits (d) None of these 1

(vii) The value of $\beta(m, n)$ is equal to :

(a) $\Gamma(m)\Gamma(n)$ (b) $\Gamma(m)/\Gamma(n)$

(c) $\beta(n, m)$ (d) None of these 1

(viii) The improper integral $\int_0^1 x^{m-1}(1-x)^{n-1}dx$, m, n > 0 is called :

(a) Alpha Function (b) Beta Function

(c) Gamma Function (d) None of these 1

(ix) The value of $\int_0^1\int_0^2 dxdy$ is :

(a) 1 (b) 2

(c) 3 (d) 0 1

(x) The value of $\int_0^1\int_0^1\int_0^1 dz\,dy\,dx$ is :

(a) 0 (b) 1

(c) 2 (d) 3 1

UNIT—I

2\. (a) Show that the sequence $<s_n>$, $s_n = \frac{1}{1.2} + \frac{1}{2.3} + \frac{1}{3.4} + \ldots + \frac{1}{n(n+1)}$ is monotonic and bounded. 3

(b) Prove that if limit of sequence $<s_n>$ exists then it is unique. 4

(c) Evaluate $\lim_{n \to \infty} \frac{1+3+5+\ldots+(2n-1)}{2n^2+1}$. 3

3\. (a) If the sequence $<s_n>$ is monotone increasing and bounded above then prove that it converges to its supremum. 4

(b) Show that the sequence <.3, .33, .333, ...> is monotonic increasing and bounded above and converges to 1/3. 3

(c) Show that the sequence $<s_n>$, where $s_n = \frac{1}{n}$ is a Cauchy Sequence. 3

UNIT—II

4. (a) Using integral test, test the convergence of $\sum_{n=1}^{\infty} \frac{1}{(n+3)(n+4)}$. 3

(b) Discuss the convergence of the series $\sum \frac{n+1}{2n^2+3}$. 3

(c) Let $\sum_{n=1}^{\infty} a_n$ be a sequence of real numbers such that $\ell = \lim_{n \to \infty} \sqrt[n]{a_n}$, $a_n \geq 0 \ \forall \ n$ then prove that :

(i) $\sum_{n=1}^{\infty} a_n$ converges if $\ell < 1$ (ii) $\sum_{n=1}^{\infty} a_n$ diverges if $\ell > 1$. 4

5. (a) Test the convergence of the series $\sum \frac{n^3 + a}{2^n + a}$. 3

(b) Using comparison test, test the convergence of the series :

$$1 + \frac{1}{2^2} + \frac{2^2}{3^3} + \frac{3^3}{4^3} + \ldots$$ 3

(c) Prove that the geometric series $\sum_{n=1}^{\infty} x^{n-1}$ converges to $\frac{1}{1-x}$ for $0 < x < 1$ and diverges for $x \geq 1$. 4

UNIT—III

6. (a) Expand $x^3 + y^3 - 3xy$ in powers of $(x - 2)$ and $(y - 3)$. 3

(b) Using $\in - \delta$ definition of a limit of a function, prove that $\lim_{(x, y) \to (1, 1)} (x^2 + 2y) = 3$. 3

(c) If f, g, are continuous at p_0 then prove that $f - g$ is continuous at p_0. 4

7. (a) If $x = \rho \cos \phi$, $y = \rho \sin \phi$, $z = z$. Find $\frac{\partial(x, y, z)}{\partial(\rho, \phi, z)}$. 3

(b) A rectangular box open at the top is to have a volume of 32 cc. Find the dimensions of the box requiring least material for its construction. 4

(c) By Lagrange's multipliers method find absolute maximum or minimum for $f(x, y) = x^2 + y^2$ where $x^4 + y^4 = 1$. 3

UNIT—IV

8. (a) Prove that $\Gamma(n+1) = n\Gamma(n)$. 4

(b) Evaluate $\int_0^\infty \frac{x^3}{(1+x)^7}\, dx$. 3

(c) Evaluate $\int_0^{\log 8} \int_0^{\log y} e^{x+y}\, dx\, dy$. 3

9. (a) Evaluate $\int_0^1 \int_0^{\sqrt{1+x^2}} \frac{1}{1+x^2+y^2}\, dx\, dy$. 3

(b) Prove that $\int_0^\infty e^{-x^{\frac{1}{n}}}\, dx = n\sqrt{n}$. 3

(c) Prove that $B(m, n) = \frac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}$. 4

UNIT—V

10. (a) Change the order of integral $\int_0^4 \int_0^{\sqrt{4x-x^2}} f(x, y)\, dy dx$. 5

(b) Evaluate by changing the order of integration $\int_0^1 \int_x^{\sqrt{2-x^2}} \frac{x\, dy\, dx}{\sqrt{x^2+y^2}}$. 5

11. (a) Evaluate by changing to polar coordinates $\iint_R \frac{x^2}{\sqrt{x^2+y^2}}\, dx\, dy$, where R is the region bounded by $0 \le x \le y$, $0 \le x \le a$. 5

(b) Evaluate $\int_0^1 \int_{x^2}^{2-x} xy\, dy\, dx$ by changing the order of integration. 5