B.Sc. (Part–II) Semester–III Examination
MATHEMATICS
(Advanced Calculus)
Paper—V

Time : Three Hours] [Maximum Marks : 60

Note :— (1) Question No. **1** is compulsory. Attempt once.

(2) Attempt **ONE** question from each unit.

1. Choose the correct alternative :—

(1) Every Cauchy sequence of real number is __________.

(a) unbounded (b) bounded

(c) bounded as well as unbounded (d) None of these

(2) The sequence $<s_n>$ where $s_n = \frac{n}{n+1}$ is _________.

(a) monotonically increasing (b) monotonically decreasing

(c) constant sequence (d) None of these

(3) The harmonic series $\Sigma \frac{1}{n}$ is _________.

(a) Convergent (b) Oscillatory

(c) Divergent (d) None of these

(4) Let Σa_n be a series with positive terms and $\lim_{n\to\infty} a_n^{1/n} = l$, then the series Σa_n is convergent if _______.

(a) $l = 1$ (b) $l > 1$

(c) $l < 1$ (d) None of these

(5) If $\lim_{P\to P_0} f(P) = f(P_0)$; where P, $P_0 \in R^2$ then ________.

(a) f is discontinuous at P_0 (b) f is continuous at P_0

(c) f is continuous at P (d) None of these

(6) If $\lim_{(x,y)\to(x_0,y_0)} f(x, y) = l$ exist then repeated limits are ________.

(a) equal (b) not equal

(c) not exist (d) None of these

(7) The function f(P) has absolute minima at P_0 in D if _________.

(a) $f(P) \leq f(P_0)$; $\forall$ $P \in D$ (b) $f(P) \geq f(P_0)$; $\forall$ $P \in D$

(c) $f(P) = f(P_0)$; $\forall$ $P \in D$ (d) None of these

(8) If $u = 2x - y$ and $v = x + 4y$ then J^{1} = ________.

(a) 1 (b) 9

(c) $\frac{1}{9}$ (d) None of these

(9) The value of $\int_1^2 \int_1^3 x^2 y \, dy \, dx$ is ________.

(a) –1 (b) $\frac{3}{28}$

(c) $\frac{28}{3}$ (d) 1

(10) The value of $\int_0^1 \int_0^1 \int_0^1 dx \, dy \, dz$ is ________.

(a) 0 (b) 2

(c) –1 (d) 1 10

UNIT—I

2. (a) Prove that a convergent sequence of a real numbers is bounded. 5

(b) Show that the sequence $<S_r>$, $S_n = \frac{1}{1!} + \frac{1}{2!} + ... + \frac{1}{n!}$ is convergent. 5

3. (p) Prove that every convergent sequence of real numbers is a Cauchy sequence. 5

(q) Show that the sequence $<S_n>$, where $S_n = \left(1 + \frac{1}{n}\right)^n$, is convergent and that $\lim_{n \to \infty} \left(1 + \frac{1}{n}\right)^n$ lies between 2 and 3. 5

UNIT—II

4. (a) Prove that the series Σx_n converges if and only if for every $\in > 0$, $\exists$ a $M(\in) \in N$ such that $m \geq n \geq M \Rightarrow | x_{n+1} + x_{n+2} + ... + x_m | < \in$. 5

(b) Test the convergence of the series $\frac{1}{x(x+2)} + \frac{1}{(x+2)(x+4)} + \frac{1}{(x+4)(x+6)} + ...$, $x \in R$, $x \neq 0$. 5

5. (p) Prove that p-series $\Sigma \frac{1}{n^p}$ is convergent for $p > 1$ and divergent for $p \leq 1$. 6

(q) Test the convergence of the series $\Sigma \frac{n^3 + a}{2^n + a} \; \forall n \in N$. 4

UNIT—III

6. (a) Prove that $\lim_{(x,y)\to(4,-1)} (3x-2y) = 14$ by using $\in - \delta$ definition of a limit of a function. 4

(b) Expand $x^3 + y^3 - 3xy$ in powers of $x - 2$ and $y - 3$. 3

(c) Let real valued functions f and g be continuous in an open set $D \subseteq R^2$ then prove that f + g is continuous in D. 3

7. (p) Prove that the function $f(x, y) = x + y$ is continuous $\forall$ $(x, y) \in R^2$. 4

(q) Expand e^{xy} at the point (2, 1) upto first three terms. 3

(r) Let $f(x, y) = \frac{xy}{x^2 - y^2}$, show that the simultaneous limit does not exist at the origin in spite of the fact that the repeated limits exist at the origin and each equals to zero. 3

UNIT—IV

8. (a) Find the maximum and minimum values of $x^3 + y^3 - 3axy$. 5

(b) Find the least distance of the origin from the plane $x - 2y + 2z = 9$ by using Lagrange's method of multipliers. 5

9. (p) If x, y are differentiable functions of u, v and u, v are differentiable functions of r, s then prove that

$$\frac{\partial(x,y)}{\partial(u,v)} \frac{\partial(u,v)}{\partial(r,s)} = \frac{\partial(x,y)}{\partial(r,s)}.$$ 5

(q) If $xu = yz$, $yv = xz$ and $zw = xy$ find the value of $\frac{\partial(x,y,z)}{\partial(u,v,w)}$. 5

UNIT—V

10. (a) Evaluate by changing the order of integration :

$$\int_0^1 \int_x^{\sqrt{2-x^2}} \frac{x}{\sqrt{x^2+y^2}}\, dy\, dx.$$ 5

(b) Evaluate $\int_v (2x+y) dv$, where v is the closed region bounded by the cylinder $z = 4 - x^2$ and the planes $x = 0$, $x = 2$, $y = 0$, $y = 2$, $z = 0$. 5

11. (p) Evaluate by Stoke's theorem $\int_c (e^x dx + 2y dy - dz)$, where c is the curve $x^2 + y^2 = 4$, $z = 2$. 5

(q) Evaluate by Gauss Divergence theorem $\iint_s \bar{f}.\bar{n}\, ds$; where

$\bar{f} = (x^2 - yz)i + (y^2 - zx)j + (z^2 - xy)k$ and s is the surface of rectangular parallelepiped $0 \le x \le a$; $0 \le y \le b$; $0 \le z \le c$. 5