http://www.sgbauonline.com/

B.Sc. (Part—II) Semester—III Examination

MATHEMATICS

(Advanced Calculus)

Paper—V

Time : Three Hours] [Maximum Marks : 60

Note :— (1) Question No. 1 is compulsory, attempt once.

(2) Attempt **ONE** question from each unit.

1. Choose the correct alternative :

(i) Every Cauchy sequence is :

(a) Unbounded (b) Bounded

(c) Oscillatory (d) None of these

(ii) The value of $\lim_{n \to \infty} \frac{4 + 3.10^n}{5 + 3.10^n}$ is :

(a) 4/5 (b) 0

(c) 4 (d) 1

(iii) If $\lim_{n \to \infty} a_n \neq 0$ then the series Σa_n is :

(a) Convergent (b) Divergent

(c) Oscillatory (d) None of these

(iv) Let Σa_n be a series of positive terms such that $\lim_{n \to \infty} \sqrt[n]{a_n} = \ell$; $\forall n$. Then Σa_n is convergent if :

(a) $\ell = 1$ (b) $\ell > 1$

(c) $\ell = 0$ (d) $\ell < 1$

(v) If $\lim_{(x, y) \to (x_0, y_0)} f(x, y) \neq f(x_0, y_0)$ then :

(a) f is continuous (b) f is continuous at (x_0, y_0)

(c) f is discontinuous (d) None of these

(vi) If $\lim_{(x, y) \to (x_0, y_0)} f(x, y) = \ell$ then the iterated limits are :

(a) Equal to ℓ (b) Greater than ℓ

(c) Less than ℓ (d) None of these

(vii) If u = 2x − y and v = x + 4y, then $\frac{\partial(u, v)}{\partial(x, y)}$ is :

(a) 7 (b) 8

(c) 1/8 (d) 9

http://www.sgbauonline.com/

(viii) The necessary condition for the extremum of f(P) at $P_0 \in D$ is :

(a) $f_x(P_0) = 0$ (b) $f_y(P_0) = 0$

(c) $f_x(P_0) = 0$ and $f_y(P_0) = 0$ (d) $f_x(P_0) = 0$ or $f_y(P_0) = 0$

(ix) The unit normal vector $\vec{n}$ to the surface $\phi(x, y, z) = 0$ is given by :

(a) $\frac{\nabla\phi}{|\nabla\phi|}$ (b) $\nabla\phi$

(c) $\vec{k}$ (d) $\vec{j}$

(x) The value of $\int_0^2\int_0^2\int_0^2 dxdydz$ is :

(a) 6 (b) 8

(c) 4 (d) 2 10

UNIT—I

2. (a) Show that the sequence $<S_n>$ where $S_n = (1 + 1/n)^n$ is convergent and its limit lies in between 2 and 3. 5

(b) Prove that every Cauchy sequence of real numbers is bounded. 3

(c) Prove that $\lim_{n\to\infty} \frac{1+3+5 \ldots + (2n-1)}{n^2} = 1$. 2

3. (p) Prove that every monotonic sequence is convergent if and only if it is bounded. 4

(q) Prove that every convergent sequence of real numbers is a Cauchy sequence. 3

(r) Show that the sequence .2, .22, .222, .2222, ... is monotonic increasing and it will converge to 2/9. 3

UNIT—II

4. (a) Prove that the series $\sum_{n=1}^{\infty} \frac{1}{n^p}$ is convergent for p > 1 and diverges when p = 1. 4

(b) Test the convergence of the series $1 - \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{3}} - \frac{1}{\sqrt{4}} + \ldots\ldots$ 3

(c) Discuss the convergence of the series $\sum_{n=1}^{\infty} \frac{n}{n^2+1}$. 3

5. (p) Let Σa_n be a series of positive terms such that $\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = \ell$. Then show that $\sum_{n=1}^{\infty} a_n$ is convergent if $\ell < 1$ and diverges when $\ell > 1$. 4

(q) Test the converges of the series $\sum_{n=1}^{\infty} \frac{1}{n(\log n)^p}$. 3

(r) Discuss the convergence of the series $\sum_{n=1}^{\infty} \frac{1}{n!}$. 3

http://www.sgbauonline.com/

UNIT—III

6. (a) Using $\in - \delta$ definition of continuity prove that $f(x, y) = x \cdot y$ is continuous for all (x, y) in xy-plane. 4

(b) Obtain the expansion of $f(x, y) = x^2 - y^2 + 3xy$ at the point (1, 2). 3

(c) Using $\in - \delta$ definition, prove that $\lim_{(x, y) \to (1, 2)} (x^2 + 3y) = 7$. 3

7. (p) Expand $x^3 + y^3 - 3xy$ in powers of $(x - 2)$ and $(y - 3)$. 4

(q) If f(x, y) is continuous at $P_0(x_0, y_0)$ then prove that it is bounded in some nbd of $P_0(x_0, y_0)$. 3

(r) Let $f(x, y) = \frac{xy}{x^2 - y^2}$. Show that simultaneous limit does not exist at the origin in spite of the fact that the repeated limits exist at the origin. 3

UNIT—IV

8. (a) Locate all critical points and determine whether a local maximum or minimum occurs at these points of $f(x, y) = x^3 - 2x^2y - x^2 - 2y^2 - 3x$. 5

(b) Find the extreme values of $u = \frac{x}{3} + \frac{y}{4}$; subject to the condition $x^2 + y^2 = 1$. 5

9. (p) Find by using Lagrange's method of multipliers, the least distance of the origin from the plane $x - 2y + 2z = 9$. 5

(q) If $xu = yz$, $yv = xz$ and $zw = xy$ then find $\frac{\partial(x, y, z)}{\partial(u, v, w)}$. 5

UNIT—V

10. (a) Evaluate $\int_0^\infty \int_x^\infty \frac{e^{-y}}{y} \, dydx$; by changing the order of integration. 5

(b) Evaluate $\int_0^1 \int_{y^2}^1 \int_0^{1-x} x \, dzdxdy$. 5

11. (p) Verify Gauss divergence theorem for the function $\bar{f} = x^2\bar{i} + y^2\bar{j} + z^2\bar{k}$ and S is a surface of unit cube $0 \le x \le 1$, $0 \le y \le 1$, $0 \le z \le 1$. 5

(q) Verify Stoke's theorem for the function $\bar{f} = y\bar{i} + z\bar{j}$ over the plane surface $2x + 2y + z = 2$ in the first octant. 5

http://www.sgbauonline.com/

http://www.sgbauonline.com/

http://www.sgbauonline.com/