AR – 535

Third Semester B. Sc. (Part - II) Examination

(New Course)

MATHEMATICS – VI

(Elementary Number Theory)

P. Pages : 7

Time : Three Hours] [Max. Marks : 60

Note : (1) Question No. **One** is compulsory, attempt it once only.

(2) Attempt **one** question from each unit.

1. Choose the correct alternative (1 mark each):—

(i) If a and b are two integers that are not both zero, then their gcd is ———— .

(a) unique (b) not unique

(c) Prime No. (d) none of these.

(ii) If x and y are odd, then x^2+y^2 is ———— ——— .

(a) A perfect square

(b) fourth power

(c) not a perfect square

(d) none of these

(iii) For any +ve integer n, there are atleast n cosecutive ________ .

(a) prime numbers

(b) composite integers

(c) negative integers

(d) none of these

(iv) The quadratic residues of 7 are ________ .

(a) 1, 2, 3 (b) 3, 5, 6

(c) 1, 2, 4 (d) none of these.

(v) For a prime P; (P–1)! ≡ ______ (mod P) :

(a) 1 (b) –1

(c) P (d) none of these.

(vi) If P is a prime divisor of the fermat number $fn=2^{2^n}+1$, then $Op^{(2)}$ = ________ .

(a) 2^n (b) 2^{n+1}

(c) 2^{2^n} (d) 2^{n-1}

(vii) If P is an odd prime, then $\left(\frac{2}{P}\right) = 1$ if— ________ .

(a) P ≡ ± 1 (mod 8)

(b) P ≡ ± 3 (mod 8)

(c) $P \equiv 0 \pmod 8$

(d) none of these.

(viii) The number of residues ———— the number of non residues.

(a) equals (b) not equals

(c) greater than (d) less than

(ix) For $n > 2$, $\phi(n)$ is an ———— .

(a) prime (b) odd integer

(c) even integer (d) none of these.

(x) The product of any m consecutive integers is divisible by ———— .

(a) $(m-1)!$ (b) $(m+1)!$

(c) m (d) m! 10

UNIT I

2. (a) If a and b are integers such that $b > 0$. Then prove that there are unique integers q and r such that $a=bq+r$ with $0 \leq r < b$. 5

(b) Using the Euclidean algorithm, find the gcd of d of the numbers 1109 and 4999 and then find integers x and y to satisfy $d = 1109x + 4999y$. 5

3. (p) Prove that there are no integers a, b, $n > 1$ such that $(a^n - b^n) \mid (a^n + b^n)$. 3

(q) If $(a, b)=1$, then show that $(a+b, a-b) = 1$ or 2. 3

(r) If c is any common multiple of a and b, then show that $[a, b] \mid c$. 4

UNIT II

4. (a) Prove that, the number of Primes is infinite. 3

(b) Find the solution of the linear Diaphantine equation $10x+6y=110$. 4

(c) Show that, for all positive integers n, $F_0F_1 \text{------} f_{n-1} = f_n - 2$ 3

5. (P) If a and b are relatively prime integers and d is a positive divisor of ab, then show that there is a unique pair of positive divisors d_1 of a and d_2 of b such that $d = d_1 d_2$. 4

(q) Prove that any two distinct Fermat number are relatively prime. 4

(r) Find the solution of the linear Diaphantine equation $12x + 8y = 199$. 2

UNIT III

6. (a) Find the remainder when the sum :

$1^5+2^5+3^5+$----------- $+ \ 200^5$ is divided by 4. 3

(b) Show that the congruence is an equivalence relation. 4

(c) Prove that, for a prime P, the positive integer a is its own inverse modules P iff $a \equiv 1 \pmod{P}$ or $a \equiv -1 \pmod{P}$. 3

7. (p) State and prove Chinese remainder theorem. 5

(q) Show that 41 divides $2^{20}-1$. 3

(r) Show that, for any integers a, b m_1 and m_2, $a \equiv b \pmod{m_1}$ and $a \equiv b \pmod{m_2}$ iff $a \equiv b(\text{mod } [m_1, m_2])$. 2

UNIT IV

8. (a) If f is a multiplicative function then show that the arithmatic function :

$f(n) = \sum_{d/n} f(d)$ is also multiplicative. 5

(b) If $n = p_1^{\alpha_1} \; p_2^{\alpha_2} \text{ ---------- } p_m^{\alpha_m}$ then

Prove that,

$\tau(n) = (\alpha_1+1)\,(\alpha_2+1) \text{ -------- } (\alpha_m+1)$

and

$$6(n) = \frac{P_1^{\alpha_1+1}-1}{P_1-1} \cdot \frac{P_2^{\alpha_2+1}}{P_2-1} \cdots\cdots \frac{P_m^{\alpha_m+1}-1}{p_m-1} \; .$$ 5

9. (p) If m is a positive integer and a is an integer with (a, m) =1 then prove that $a^{\phi(m)} \equiv 1 (\text{mod } m)$. 5

(q) Prove that the mobius μ–function is multiplicative. 5

UNIT V

10. (a) If P is a prime and

$f(x) = a_n x^n + \text{----} + a_1 x + a_0$ is a polynomial of degree $n \geq 1$ with integral coefficient and $a_0 t_p$ i.e. $a_0 \not\equiv 0$ (mod p), then prove that $f(x) \equiv 0$ (mod P) has at most n incongruent solutions modulo P. 5

(b) If P is an odd prime and a, b are integers with (a, p) = 1= (b, p) then show that :

(i) $a \equiv b \;(\text{mod } p) \Rightarrow \left(\frac{a}{P}\right) = \left(\frac{b}{P}\right)$

(ii) $\left(\frac{a}{P}\right)\left(\frac{b}{P}\right) = \left(\frac{ab}{P}\right)$

(iii) $\left(\frac{a^2}{p}\right) = 1$ 5

11. (p) Solve the quadratic congruence $x^2+7x+10 \equiv 0 \pmod{11}$ 3

(q) If P is a prime number and $d/_{(P-1)}$, then prove that the congruence $x^d - 1 \equiv 0 \pmod{p}$ has exactly d solutions. 4

(r) If (a, m) = d > 1, then prove that m has no primitive root a. 3