AR – 533

Third Semester B. Sc. (Examination

(N

MATHEMA... – V

(Advanced Calculus)

P. Pages : 8

Time : Three Hours] [Max. Marks : 60

Note : (1) Question No. **one** is compulsory, attempt once.

(2) Attempt **one** question from each unit.

1. Choose the correct alternative :—

(i) Let $\{ x_n \}$ be a Cauchy sequence of a real numbers. Then the sequence $\{ \cos (x_n) \}$ is

(a) unbounded

(b) bounded but not Cauchy

(c) Cauchy but not bounded

(d) Cauchy sequence. 1

(ii) The series $\sum x_n = \sum n^2$ is .....

(a) convergent

(b) divergent

(c) oscillatory

(d) none of these. 1

(iii) If L nen the series $\sum x_n$ is
n –

(a)

(b)

(c) (II – Part

(d) ese. 1

(iv) $\int_{0}^{1}$ = ...
0 1

(a) $\frac{12}{3}$

(b) $\frac{13}{3}$

(c) $\frac{14}{3}$

(d) $\frac{15}{3}$. 1

(v) A bounded sequence in R

(a) must have at least two limits

(b) have a convergent subsequence

(c) has exactly one limit point

(d) None of these. 1

(vi) The P–series where P=1, given by

$\sum \frac{1}{n} = 1 + \frac{1}{2} + \frac{1}{3} + ... + \frac{1}{n} + ...$ is called ..... series

(a) geometric series

(b) arithmatic

(c) harmomic

(d) convergent. 1

(vii) The critical points of f (x,y) are given by .....

(a) $f_x = f_y$

(b) $f_x = 0$

(c) $f_y = 0$

(d) $f_x = 0$, $f_y = 0$. 1

(viii) The value of $\int_1^2 \int_1^3 dx\, dy$ is

(a) 1

(b) 2

(c) 3

(d) 4. 1

(ix) If iterated limits of function are not equal at point then .....

(a) limit exist at point

(b) limit does not exist

(c) limit is zero

(d) none of these. 1

(x) A function f (p) is said to have absolute minimum at $P_0 \in D$ iff for all $P \in D$ satisfies the condition.

(a) $f(P_0) \geq f(P)$

(b) $f(P_0) \leq f(P)$

(c) $f(P_0) = f(P)$

(d) none of these. 1

UNIT – I

2. (a) Evaluate $\lim_{n \to \infty} \left[\frac{2+3.10^n}{3+4.10^n} \right]$ 2

(b) Show that the sequence $< S_n >$ defined by

$$S_n = \frac{1}{3+1} + \frac{1}{3^2+1} + \ldots\ldots + \frac{1}{3^n+1}$$

is monotonic and bounded. 4

(c) Define Cauchy sequence and prove that every convergent sequence of real numbers is a Cauchy sequence. 4

3. (p) If $<S_n>$, $<t_n>$ and $<u_n>$ be three sequences such that

(i) $S_n \leq t_n \leq u_n \; \forall \; n$ and

(ii) $\lim_{n \to \infty} S_n = \lim_{n \to \infty} u_n = l$, then prove that

$\lim_{n \to \infty} t_n = l.$ 4

(q) Show that the sequence $< S_n >$ defined by $S_n = 1 + \frac{1}{2} + \frac{1}{3} + \ldots + \frac{1}{n}$ does not onverges. 4

(r) Define :

(i) Convergent sequence,

(ii) Monotone sequence. 2

UNIT – II

4. (a) A geometric series $\sum_{n=1}^{\infty} x^{n-1}$ converges to $\frac{1}{1-x}$ for $0 < x < 1$ and diverges for $x \geq 1$ prove this. 4

(b) Test the convergence of the series

(i) $\sum \left(\frac{n}{n+1}\right)^{n^2}$

(ii) $1 - \frac{1}{3 \times 2^2} + \frac{1}{5 \times 3^2} - \frac{1}{7 \times 4^2} + \ldots$

3 + 3

5. (p) Let $\sum x_n$ be a positive term series such that $\lim_{n \to \infty} \frac{x_{n+1}}{x_n} = l$. Then prove that $\sum x_n$ is convergent if $l < 1$. 4

(q) Test the convergence of the series

(i) $\frac{1}{1.2} + \frac{1}{2.3} + \frac{1}{3.4} + ...$

(ii) $\sum \frac{n^3 + a}{2^n + a}$

3 + 3

UNIT – III

6. (a) Prove that the function f (x , y) = x + y is continuous $\forall$ (x, y) $\in R^2$. 4

(b) Using $\epsilon - \delta$ definition of a limit of a function, prove that

$$\lim_{(x,y) \to (1,1)} (x^2 + 2y) = 3.$$

3

(c) Expand e^{xy} at the point (2, 1) up to first three terms. 3

7. (p) Let f (x , y) be defined and continuous in the open region D and let $f(x_1, y_1) = Z_1$, $f(x_2, y_2) = Z_2$, $Z_1 \neq Z_2$. Then prove that for every number Z_0 between Z_1 and Z_2, there is a point (x_0, y_0) of D for which $f(x_0, y_0) = Z_0$. 4

(q) Let $Z = f(x, y) = \dfrac{y(x^2 + y^2)}{y^2 + (x^2 + y^2)^2}$. Show that f has limit 0 as $(x, y) \rightarrow (0, 0)$ on a ray $x = at$, $y = bt$, but f does not have limit 0 as $(x, y) \rightarrow (0, 0)$ along $y = x^2$. 4

(r) Define

(i) Continuity ($\epsilon - \delta$ definition)

(ii) Continuity on an interval. 2

UNIT – IV

8. (a) A rectangular box open at the top is to have a volume of 32 CC. Find the dimension of the box requiring least material for its construction. 5

(b) Find by using Lagrange's method of multipliers, the least distance of the origin from the plane $x - 2y + 2z = 9$. 5

9. (p) If $x = r \cos \theta$, $y = r \sin \theta$,

find $J = \dfrac{\partial(x, y)}{\partial(r, \theta)}$ and $J' = \dfrac{\partial(r, \theta)}{\partial(x, y)}$ and

prove that $JJ' = 1$. 5

(q) Find the maximum and minimum values of $x^3 + y^3 - 3axy$. 5

UNIT – V

10. (a) Evaluate $\int_0^1 \int_0^{1-x} \int_0^{1-x-y} dx\, dy\, dz$. 5

(b) Evaluate the following integral by changing the order of integration

$$\int_0^{\infty} \int_x^{\infty} \frac{e^{-y}}{y}\, dy\, dz$$ 5

11. (a) Verify Stokes theorem for the vector field defined by $\overline{f} = (x^2 - y^2)\overline{i} + 2xy\overline{j}$ in the rectangular region in the xy – plane bounded by lines $x = 0$, $x = a$, $y = 0$, $y = b$. 5

(b) Evaluate $\iint_S \overline{F}.\overline{n}\, ds$ where $\overline{F} = x^2\overline{i} + y^2 j + z^2 k$ and S is the surface of the solid cut off by the plane $x + y + z = a$ from the first octant, by Gauss divergence theorem. 5