AR – 532

Third Semester B. Sc. (Part – II) Examination
(Old)

MATHEMATICS - V
(Advanced Calculus)

P. Pages : 8

Time : Three Hours] [Max. Marks : 60

Note : (1) Question no. **one** is compulsory.
(2) Attempt **one** question from each unit.

1. Choose the correct alternatives :—

(i) The sequence <n> is ....

(a) Monotonic decreasing

(b) Monotonic increasing

(c) Constant sequence

(d) None of the above 1

(ii) If $\lim S_n = 1$ and $S_n \geq a \ \forall\ n \in N$, then

(a) $l < a$

(b) $l = 0$

(c) $l \geq a$

(d) $l > \frac{1}{a}$ 1

(iii) The series $\Sigma \frac{1}{n}$ is

(a) Convergent

(b) Divergent

(c) Bounded

(d) Constant. 1

(iv) Cauchy's root test fails if

(a) $\lim_{n \to \infty} \sqrt[n]{a_n} = 1$

(b) $\lim_{n \to \infty} \sqrt[n]{a_n} > 1$

(c) $\lim_{n \to \infty} \sqrt[n]{a_n} < 1$

(d) None of these. 1

(v) If iterated limits of a function are not equal at point then

(a) Limit exist at point

(b) Limit does not exist

(c) Limit is zero

(d) None of these 1

(vi) The necessary condition for the extremum of f(x, y) is

(a) $f_x = 0 \; ; \; f_y \neq 0$

(b) $f_x \neq 0 \; ; \; f_y = 0$

(c) $f_x = 0$; $f_y = 0$

(d) None of these 1

(vii) $\Gamma(1) = ?$

(a) 0

(b) 1

(c) –1

(d) n 1

(viii) The value of $\beta(5, 3)$ is :

(a) 105

(b) –105

(c) $\frac{1}{105}$

(d) $-\frac{1}{105}$ 1

(ix) Under the transformation of $\iint_R f(x, y)\, dxdy$ into polar coordinates the value of dxdy is :

(a) $r dr d\theta$

(b) $dr d\theta$

(c) $r^2 dr d\theta$

(d) $\frac{1}{r} dr d\theta$ 1

(x) The value of $\int_0^1 \int_0^1 dxdy$ is :

(a) 2

(b) 3

(c) 1

(d) None of these. 1

UNIT I

2. (a) Show that the sequence <·5, ·55, ·555, ...> is monotonic increasing and bounded above. 3

(b) If $<S_n>$ and $<t_n>$ be sequences such that $\lim_{n\to\infty} S_n = 1$ and $\lim_{n\to\infty} t_n = m$ then prove that $\lim_{n\to\infty} (S_n - t_n) = l - m$. 4

(c) Show that Cauchy sequence is bounded. 3

3. (a) Show that the sequence $<S_n>$, where $S_n = \frac{1}{1\cdot 2} + \frac{1}{2\cdot 2^2} + \frac{1}{3\cdot 2^3} + \dots + \frac{1}{n\cdot 2^n}$ is convergent 3

(b) Show that the sequence $<S_n>$, where $S_n = \frac{n}{n+1}$ is a Cauchy sequence. 3

(c) Prove that the sequence $<S_n>$, where $S_n = \frac{1}{n+1} + \frac{1}{n+2} + \dots + \frac{1}{n+n}$ is monotonic and bounded. 3

UNIT II

4. (a) Examine the convergence of series $\sum n^2 e^{-n^3}$. 3

(b) Prove that the geometric series $\sum_{n\to 1}^{\infty} x^{n-1}$ converges to $\frac{1}{1-x}$ for $0 < x < 1$ and diverges for $x \geq 1$. 4

(c) Test the convergence of the series :

$$\Sum \left(\frac{n}{n+1}\right)^{n^2}.$$ 3

5. (a) Test the convergence of the series :

$$\frac{4}{7} x + \frac{7}{11} x^2 + \frac{10}{15} x^3 + \frac{13}{19} x^4 + ...$$ 4

(b) Test the convergence of the series

$$\Sigma \frac{n!}{n^n}$$ 3

(c) Examine the convergence of the series :

$$\Sigma \frac{1}{n^{1+\frac{1}{n}}}$$ 3

UNIT III

6. (a) Prove that product of two continuous functions is continuous. 4

(b) Expand x^2-y^2+3xy in power of $(x-1)$ and $(y-2)$. 3

(c) If $x = r\sin\theta\cos\phi$ $y = r\sin\theta\sin\phi$ $z = r\cos\theta$ then find $\frac{\partial(x, y, z)}{\partial(r, \theta, \phi)}$ 3

7. (a) Find the points on the surface $z^2 = xy + 1$ at a least distance from the origin. 3

(b) If x, y are differentiable functions of u, v and u, v are differentiable functions of r, s then prove that

$$\frac{\partial(x, y)}{\partial(u, v)} \cdot \frac{\partial(u, v)}{\partial(r, s)} = \frac{\partial(x, y)}{\partial(r, s)}.$$ 4

(c) By Lagranges multipliers method find absolute maximum or minimum for function $f(x, y) = xy + yz + zx$, where $x^2 + y^2 + z^2 = 1$ 3

UNIT IV

8. (a) Prove that

$$\beta(m, n) = \int_0^1 \frac{x^{m-1} + x^{n-1}}{(1+x)^{m+n}} dx$$ 4

(b) Prove that

$$\int_0^{\infty} x^{n-1} e^{-kx} dx = \frac{\Gamma(n)}{k^n}$$, where n, k>0 are constants. 3

(c) Evaluate

$$\int_0^1 \int_0^{\sqrt{1+x^2}} \frac{1}{1+x^2+y^2} \, dx\, dy.$$ 3

9. (a) Prove that $\int_0^{\infty} e^{-x^{\frac{1}{n}}} dx = n\,\Gamma(n)$. 3

(b) Define Beta function. Prove that $\beta(m, n) = \beta(n, m)$. 1+3

(c) Evaluate $\int_{-2}^{2} dy \int_{y^2-1}^{3} (x+2y)\, dx$. 3

UNIT V

10. (a) Change the order of integral

$$\int_0^{2a} \int_{x^2/4a}^{3a-x} f(x, y)\, dx\, dy.$$ 5

(b) Evaluate by changing the order of integration

$$\int_0^1 \int_x^{\sqrt{2-x^2}} \frac{x\, dy\, dx}{\sqrt{x^2+y^2}}$$ 5

11. (a) Evaluate $\int_0^a \int_y^a \frac{x^2\,dx\,dy}{(x^2+y^2)^{3/2}}$ by changing into polar coordinates. 5

(b) Evaluate the following integral by changing the order of integration

$\int_0^b \int_{y-b}^{2y} xy\,dx\,dy.$ 5