## UNIT – V

10. (a) Two spheres of radii $r_1$ and $r_2$ cut orthogonally. Prove that the radius of the Common Circle is $\frac{r_1 r_2}{\sqrt{r_1^2 + r_2^2}}$ 5

(b) Find the equation of the right circular cylinder which passes through the circle $x^2+y^2+z^2=9$ , $x-y+z=3$. 5

11. (c) Find the equation of the right circular cone whose vertex is $(2,-3,5)$, axis makes equal angles with the coordinate axes and semi-vertical angle is $30^0$. 4

(d) Show that the plane $2x-2y+z+12=0$ touches the sphere $x^2+y^2+z^2-2x-4y+2z-3=0$ and find the point of contact. 3

(e) Find the equation of sphere which passes through points $(1,0,0)$ $(0,1,0)$ and $(0,0,1)$ and radius as small as possible. 3







# Second Semester B. Sc. (Part – I) Examination

(Old Course)

## 2S – MATHEMATICS

Paper – IV

(Vector Analysis and Geometry)

P. Pages : 8

Time : Three Hours] [Max. Marks : 60

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**Note :** (1) Question No. **one** is compulsory. Solve it in one attempt only.

(2) Solve **one** question from each unit.

1. Choose the correct alternatives of the following :—

(i) $[\, i - j \, , \, j \, , \, i \,]$ is equal to .....

(a) 0

(b) i

(c) j

(d) k 1

(ii) Three vectors $\bar{a}, \bar{b}, \bar{c}$ are coplaner iff .....

(a) $\bar{a} \times (\bar{b} \times \bar{c}) = \bar{0}$

(b) $\bar{a} \circ (\bar{b} \times \bar{c}) = 0$

(c) $(\bar{a} \circ \bar{b}) \times \bar{c} = 0$

(d) $(\bar{a} \times \bar{b}) \times \bar{c} = \bar{0}$ 1

(iii) If $\overline{r} = x^2i + y^2j$ then the value of $\nabla o \overline{f}$ at (1,1) is .....

(a) 4

(b) −4

(c) 2

(d) −2 1

(iv) If $\overline{r} = xi + yj + zk$ then div $\overline{r}$ is .....

(a) 1

(b) 0

(c) 3

(d) None of these 1

(v) A curve is helix if .....

(a) $\overline{e}$ o $\overline{t}$ = cos α

(b) $\overline{e}$ o $\overline{t}$ = tan α

(c) $\overline{e}$ x $\overline{t}$ = cos α

(d) $\overline{e}$ x $\overline{t}$ = tan α 1

(vi) A plane passing through point P on curve and containing the tangent and normal at P is called .....

(a) rectifying plane

(b) normal plane



(e) If $\overline{f} = xy^2i + 2x^2yzj - 3yz^2k$ then find div $\overline{f}$ at point (1, −1, 1). 2

## UNIT - IV

8. (a) Find the equation of plane through (−1, 3, 2) and perpendicular to each of planer, $x+2y+3z=5$ and $3x+3y+z=9$. 4

(b) Prove that the line $\frac{x-x_1}{l} = \frac{y-y_1}{m} = \frac{z-z_1}{n}$ lies in a plane $ax + by + cz + d = 0$ if $al + bm + cn = 0$ and $ax_1 + by_1 + cz_1 + d = 0$. 3

(c) Find the equation of plane through the intersection of the planer $x + 3y + 6 = 0$ and $3x - y - 4z = 0$, whose perpendicular distance from the origin is unity. 3

9. (d) A variable plane is at a constant distance P from the origin and meets the axes in A, B, C. Show that the locus of the centroid of tetrahedron OABC is $x^{-2} + y^{-2} + z^{-2} = 16^{-2}P$. 5

(e) Show that the plane through (α, β, γ) parallel to $ax + by + cz + d = 0$ is $a(x - \alpha) + b(y - \beta) + c(z - \gamma) = 0$. 5

## UNIT – III

6. (a) Evaluate the line integral $\int_C \bar{F}\cdot d\bar{r}$ for $\bar{F} = xy\,i + y^2\,j$ where C is curve in xy–plane $y = 2x^2$ from (1,2) to (2,8). 5

(b) Find the work done in moving a particle along the parabola $y^2 = x$ in the xy–plane from (0,0) to (1,1) if the force field is given by $\bar{F} = (2x+y-7z)\,i + (7x-2y+2z^2)\,j + (3x-2y+4z^2)\,k$. 5

7. (c) Let R be a closed bounded region in the X–Y plane whose boundary is a simple closed curve C, which may be cut by any line parallel to the co–ordinate axes in at most two points. Let M (x,y) and N (x,y) be functions that are continuous and have continuous partial derivatives $\frac{\partial M}{\partial y}$ and $\frac{\partial N}{\partial x}$ in R.

Then prove that :

$$\iint_C \left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}\right) dx\,dy = \oint_C (M dx + N dy)$$ 5

(d) If $\bar{f} = zi + xj + yk$ then show that curl (curl $\bar{f}$) = $\bar{0}$. 3



(c) oscalating plane

(d) radical plane 1

(vii) The equations $ax + by + cz + d = 0$, $a_1x + b_1y + c_1z + d_1 = 0$ together represent a .....

(a) plane

(b) straight line

(c) sphere

(d) point 1

(viii) Pair of lines represented by $ax^2 + by^2 + cz^2 + 2hxy + 2gzx + 2fyz = 0$ will be at right angles if .....

(a) $a = b + c$

(b) $a + b + c = 1$

(c) $a + b + c = 0$

(d) $a = 0$ 1

(ix) The equation of cone with vertex at origin is .....

(a) linear

(b) cubic

(c) homogeneous

(d) quadratic 1

(x) Any second degree equation in which coefficients of $x^2, y^2, z^2$ are equal and terms xy, yz, zx are absent represents a .....

(a) cone

(b) cylinder

(c) sphere

(d) plane 1

## UNIT - I

2. (a) Prove that
$\bar{u} \times (\bar{v} \times \bar{w}) = (\bar{u} \circ \bar{w})\, \bar{v} - (\bar{u} \circ \bar{v})\, \bar{w}$. 4

(b) If $\bar{A} = x^2yzi - 2xz^3j + xz^2k$ and $\bar{B} = 2zi + yj - x^2k$ find
$$\frac{\partial^2}{\partial x\, \partial y}(\bar{A} \times \bar{B}) \text{ at } (1, 0, -2).$$ 3

(c) Prove that
$$\bar{a} = \tfrac{1}{2}[\, i \times (\bar{a} \times i) + j \times (\bar{a} \times j) + k \times (\bar{a} \times k)\,].$$ 3

3. (d) Prove that a vector function $\bar{u}$ is constant in magnitude iff
$$\bar{u} \circ \frac{d\bar{u}}{dt} = 0.$$ 4

(e) If $\bar{a} \times (\bar{b} \times \bar{c}) = (\bar{a} \times \bar{b}) \times \bar{c}$ then prove that $\bar{b}$ is perpendicular to plane of $\bar{a}$ and $\bar{c}$. 3

(f) If $\phi = 11xyz^2$ and C is the curve $x = t^2$, $y = 10t$, $z = 4t^3$ from $t = 0$ to $t = 1$. Evaluate $\int_C f\, dr$ 3

## UNIT - II

4. (a) Show that the curvature of the helix $\bar{r} = (a \cos\theta, a \sin\theta, a\theta \tan\alpha)$ is $\frac{\cos^2\alpha}{a}$ and the torsion is $\pm\frac{\sin\alpha \cos\alpha}{a}$ 5

(b) Find the equations of the normal plane and the tangent line for the twisted cubic $x = at$, $y = bt^2$, $z = ct^3$ at the point $t = 1$. 5

5. (c) State and prove Frenet–Serret formulae. 1+4

(d) If the tangent and the binormal at a point of a curve make angles $\theta$ and $\phi$ respectively with a fixed direction, show that
$$\frac{\sin\theta}{\sin\phi}\,\frac{d\theta}{d\phi} = -\frac{k}{J}$$ 5