

B. Sc. (Part-I) Semester—II Examination
MATHEMATICS
(Vector Analysis and Solid Geometry)
Paper-IV

Time : Three Hours]

[Maximum Marks : 60

Note :— (1) Question No. 1 is compulsory; attempt it once only.

(2) Attempt one question from each unit.

1. Choose the correct alternative :

(i) If three vectors \vec{a} , \vec{b} , \vec{c} are coplanar, then for scalar triple product, which of the following is correct ?(a) $\vec{b} \times \vec{c}$ is perpendicular to the vector \vec{a} (b) $\vec{b} \times \vec{c}$ is parallel to the vector \vec{a} (c) $\vec{b} \times \vec{c}$ is equal to the vector \vec{a} (d) None of these. 1

(ii) The scalar triple product represents the volume of the _____.

(a) rectangle

(b) sphere

(c) parallelepiped

(d) ellipse 1(iii) The curvature k is determined _____.

(a) only in magnitude

(b) only in sign

(c) both in magnitude and sign

(d) neither in magnitude nor sign 1(iv) A plane determined by the tangent and binormal at $P(\vec{r})$ to the curve $\vec{r} = \vec{r}(s)$ is a _____.

(a) osculating plane

(b) rectifying plane

(c) normal plane

(d) none of these 1

(v) Which of the following quantity is defined ?

(a) $\text{div} (\text{div } \vec{f})$ (b) $\text{curl} (\text{div } \vec{f})$ (c) $\text{grad} (\text{curl } \vec{f})$ (d) $\text{grad} (\text{div } \vec{f})$ 1(vi) A vector \vec{f} is solenoidal if _____.(a) $\text{curl } \vec{f} = 0$ (b) $\text{div } \vec{f} = 0$ (c) $\text{grad } \vec{f} = 0$ (d) $\text{grad} (\text{div } \vec{f}) = 0$ 1

- (vii) If the radius of the circle is equal to the radius of the sphere, the circle is called a _____.
 (a) small circle (b) imaginary circle
 (c) great circle (d) none of these 1
- (viii) The equations of the sphere and the plane taken together represent a _____.
 (a) sphere (b) plane
 (c) straight line (d) circle 1
- (ix) Every section of a right circular cone by a plane perpendicular to its axis is _____.
 (a) a sphere (b) a cone
 (c) a circle (d) a cylinder 1
- (x) The general equation of the cone passing through the coordinate axes is _____.
 (a) $fyz + gzx + hxy = 0$ (b) $yz + zx + xy = 0$
 (c) $ax^2 + by^2 + cz^2 = 0$ (d) $x^2 + y^2 + z^2 = 0$ 1

UNIT—I

2. (a) Show that $\vec{a} \times (\vec{b} \times \vec{c}), \vec{b} \times (\vec{c} \times \vec{a}), \vec{c} \times (\vec{a} \times \vec{b})$ are coplanar. 5
- (b) If $\vec{a}, \vec{b}, \vec{c}$ be three unit vectors such that $\vec{a} \times (\vec{b} \times \vec{c}) = \frac{1}{2} \vec{b}$, find the angles which \vec{a} makes with \vec{b} and \vec{c} , \vec{b} and \vec{c} being non-parallel. 5
3. (p) If \vec{f} is a vector function of t and u is a scalar function of t , then prove that :

$$\frac{d}{dt}(u\vec{f}) = u \frac{d\vec{f}}{dt} + \frac{du}{dt} \vec{f}. \quad 5$$
- (q) Evaluate $\int_1^2 \vec{r} \times \frac{d^2 \vec{r}}{dt^2} dt$, where

$$\vec{r}(t) = 5t^2 \vec{i} + t \vec{j} - t^3 \vec{k}. \quad 5$$

UNIT—II

4. (a) Prove that helices are the only twisted curves whose Darboux's vector has a constant direction. 5
- (b) For the curve $x = 3t, y = 3t^2, z = 2t^3$ at the point $t = 1$, find the equations for osculating plane, normal plane and rectifying plane. 5
5. (p) For the curve $x = a(3t - t^3), y = 3at^2, z = a(3t + t^3)$, show that the curvature and torsion are equal. 5
- (q) If $\vec{t}' = \vec{d} \times \vec{t}, \vec{n}' = \vec{d} \times \vec{n}, \vec{b}' = \vec{d} \times \vec{b}$, then find the vector \vec{d} . 5

UNIT—III

6. (a) If $\vec{r} = x\vec{i} + y\vec{j} + z\vec{k}$, then show that $\text{div}(r^n \vec{r}) = (n+3)r^n$. 4
- (b) Find the directional derivative of $\phi = xy^2 + yz^2$ at the point $(2, -1, 1)$ in the direction of the vector $\vec{i} + 2\vec{j} + 2\vec{k}$. 3
- (c) If $\phi = 3x^2y - y^3z^2$, find $\text{grad } \phi$ at the point $(1, -2, -1)$. 3
7. (p) If $\vec{F} = (2x + y^2)\vec{i} + (3y - 4x)\vec{j}$, evaluate $\int_C \vec{F} \cdot d\vec{r}$ along the parabolic arc $y = x^2$ joining $(0, 0)$ and $(1, 1)$. 5
- (q) Apply Green's theorem to prove that the area enclosed by a simple plane curve C is $\frac{1}{2} \int_C (x dy - y dx)$. Hence find the area of an ellipse whose semi-major and semi-minor axes are of lengths a and b . 3+2

UNIT—IV

8. (a) Find the equation of a sphere for which the circle $x^2 + y^2 + z^2 + 7y - 2z + 2 = 0$, $2x + 3y + 4z = 8$ is a great circle. 5
- (b) Find the equation of the sphere circumscribing the tetrahedron whose faces are :
- $$x = 0, y = 0, z = 0, \frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1. \quad 5$$
9. (p) State and prove the condition for the orthogonality of two spheres. 1+4
- (q) Find the coordinates of the centre and radius of the circle $x + 2y + 2z = 15$; $x^2 + y^2 + z^2 - 2y - 4z = 11$. 5

UNIT—V

10. (a) Find the equation of the cone whose vertex is at the point (α, β, γ) and whose generators touch the sphere $x^2 + y^2 + z^2 = a^2$. 5
- (b) Find the equation of right circular cone whose vertical angle is 90° and its axis is along the line $x = -2y = z$. 5
11. (p) Find the equation to the cylinder whose generators are parallel to the line $\frac{x}{1} = \frac{y}{-2} = \frac{z}{3}$ and the guiding curve is the ellipse $x^2 + 2y^2 = 1, z = 3$. 5
- (q) Find the equation of the right circular cylinder of radius z whose axis passes through $(1, 2, 3)$ and has direction cosines proportional to $2, -3, 6$. 5

