Second Semester B. Sc. (Part - I) Examination

(New)

MATHEMATICS

Paper – IV

Vector Analysis and Solid Geometry

P. Pages : 7

Time : Three Hours] [Max. Marks : 60

Note : (1) Question No. **1** is compulsory and attempt it once only.

(2) Solve **one** question from each unit.

1. Choose correct alternative of the following:

(i) A vector which is not null is a---------

(a) Unit vector

(b) Improper vector

(c) Proper vector

(d) None of these 1

(ii) Let $\bar{a}$ and $\bar{b}$ be any two non–zero vectors. Then $\bar{a}$ and $\bar{b}$ are orthogonal iff------

(a) $\bar{a} \cdot \bar{b} = 0$

(b) $\bar{a} \times \bar{b} = 0$

(c) $\bar{a} \cdot \bar{b} = 1$

(d) $\bar{a} \times \bar{b} = 1$ 1

(iii) Three vectors $\bar{a}, \bar{b}, \bar{c}$ are coplanar iff----

(a) $\bar{a} \times (\bar{b} \times \bar{c}) = 0$

(b) $(\bar{a} \times \bar{b}) \times \bar{c} = 0$

(c) $(\bar{a} \cdot \bar{b}) \times \bar{c} = 0$

(d) $\bar{a} \cdot (\bar{b} \times \bar{c}) = 0$ 1

(iv) If $\bar{r} = x\bar{i} + y\bar{j} + z\bar{k}$ then curl $\bar{r}$ = -----

(a) $\bar{i}$ (b) $\bar{j}$

(c) $\bar{k}$ (d) $\bar{o}$ 1

(v) For any curve $\bar{t}' \cdot \bar{b}'$ = ---------

(a) K (b) J

(c) KJ (d) –KJ 1

(vi) The equation $x^2+y^2+z^2+2ux+2vy+2wz+d=0$ represent a real sphere if

(a) $u^2 + v^2 + w^2 = d$

(b) $u^2 + u^2 + w^2 > d$

(c) $u^2 + v^2 + w^2 < d$

(d) $u^2 + v^2 + w^2 = 0$ 1

(q) Find the equation of the two spheres which pass through the circle $x^2 + y^2 + z^2 = 5$

$x + 2y + 2z = 3$ and touch $4x + 3y - 15 = 0$. 5

UNIT V

10. (a) Find the equation of the cylinder whose generators are parallel to the line $\frac{x}{1} = \frac{y}{-2} = \frac{z}{3}$ and the guiding curve is the ellipse $x^2 + 2y^2 = 1$, $z = 3$. 5

(b) Find the equation of the right circular cylinder which passes through the circle $x^2 + y^2 + z^2 = 9$, $x - y + z = 3$. 5

11. (p) Find the equation of right circular cone which has its vertex at (0, 0, 10) and whose intersection with xy--plane is circle of radius 5. 5

(q) Find the equation of right circular cone whose vertical angle is 90^0 and its axis is along the line $x = -2y = z$. 5

7. (p) State Green's theorem and evaluate $\int_c [e^{-x} \sin y \, dx + e^{-x} \cos y \, dy]$, where c is a rectangle with vertices (0, 0), (π,0) (π, π/2), (0, π/2). 5

(q) Evaluate $\int_c \bar{F}.d\bar{r}$ from (0, 0, 0) to (1, 1, 1) along the straight line joining (0, 0, 0) and (1,1,1), when $\bar{F}=(3x^2+6y)i = 14yzj + 20xz^2k$. 5

UNIT IV

8. (a) Find the equation of the sphere which passes through the point $(1,-3,4),(1,-5,2)$ and $(1,-3,0)$ whose centre lies on the plane $x + y + z = 0$. 5

(b) Prove that the two spheres.

$x^2+y^2+z^2+2u_1x+2v_1y+2w_1z+d_1=0$ and

$x^2+y^2+z^2+2u_2x+2v_2y+2w_2z+d_2=0$ will be orthogonal if $2u_1u_2+2v_1v_2+2w_1w_2=d_1+d_2$. 5

9. (p) Two spheres of radii r_1 and r_2 cut orthogonally. Prove that the radius of the common circle is $\frac{r_1 r_2}{\sqrt{r_1+r_2}}$. 5

(vii) A necessary and sufficient condition for f(t) to have constant magnitude is -----

(a) $|\bar{f}| = 0$ (b) $\bar{f} \cdot \frac{d\bar{f}}{dt} = 0$

(c) $\bar{f} \times \frac{d\bar{f}}{dt} = 0$ (d) None of these 1

(viii) Every homogeneous equation of second degree in x, y and z represents a ----, whose vertex is at the origin.

(a) Cone (b) Cylinder

(c) Sphere (d) None of these 1

(ix) A helix is twisted curve whose tangent makes a constant angle with a -----

(a) Tangent (b) Normal

(c) Fixed direction (d) Binormal 1

(x) The curve of intersection of two spheres is--------

(a) Circle (b) Point

(c) Line (d) Plane 1

UNIT I

2. (a) Show that the necessary and sufficient condition for $\bar{f}(t)$ to have constant direction is $\bar{f} \times \frac{d\bar{f}}{dt} = 0$. 4

(b) If the vectors $\bar{a}, \bar{b}, \bar{c}, \bar{d}$ are coplanar then show that $(\bar{a} \times \bar{b}) \times (\bar{c} \times \bar{d}) = 0$. 3

(c) Given that $\bar{r}(t) = \begin{cases} 2\bar{i} - \bar{j} + 2\bar{k}, & \text{for } t = 2 \\ 4\bar{i} - 2\bar{j} + 3\bar{k}, & \text{for } t = 3 \end{cases}$

Show that $\int_2^3 \bar{r} \cdot \frac{d\bar{r}}{dt}\, dt = 10$. 3

3. (P) Prove that $\bar{a} \times (\bar{b} \times \bar{c}) = (\bar{a} \cdot \bar{c})\bar{b} - (\bar{a} \cdot \bar{b})\bar{c}$. 4

(q) If $\bar{f}$ and $\bar{g}$ are vector functions of t, then prove that $\frac{d}{dt}(\bar{f} \cdot \bar{g}) = \bar{f} \cdot \frac{d\bar{g}}{dt} + \frac{d\bar{f}}{dt} \cdot \bar{g}$ 3

(r) Prove that $\bar{i} \times (\bar{a} \times \bar{i}) + \bar{j} \times (\bar{a} \times \bar{j}) + \bar{k} \times (\bar{a} \times \bar{k}) = 2\bar{a}$. 3

UNIT II

4. (a) State and prove Serret–Frenet formulae. 5

(b) Prove that the position vector of the current point on a curve satisfies the differential equation $\frac{d}{ds}\left(6\frac{d}{ds}(\varrho\vec{r}'')\right) + \frac{d}{ds}(6/\varrho\,\vec{r}') + \frac{\varrho}{6}\,\vec{r}'' = 0$. 5

5. (p) For the helix $x = a\cos\theta$, $y = a\sin\theta$, $z = a\theta\tan\alpha$; a, α are constants, show that $k = \frac{1}{a}\cos^2\alpha$, $T = \pm\frac{1}{a}\sin\alpha\cos\alpha$.

Find ϱ and 6. 5

(q) Prove that for any curve :

$[b', b'', b'''] = T^3(k'T - kY') = T^5(k/T)'$ 5

UNIT III

6. (a) Find the directional derivative of $\phi = xy^2 + yz^2$ at the point (2,–1, 1) in the direction of the vector $\bar{i} + 2\bar{j} + 2\bar{k}$. 3

(b) Prove that $r^n \bar{r}$ is irrotational. Find the value of n, when it is solenoidal. 3

(c) Find the total work done in moving a particle in a force field given by $\bar{F} = 2xy\bar{i} + 3z\bar{j} - 6x\bar{k}$ along the curve $x = t^2 + 1$ $y = t$ $z = t^3$ from $t = 0$ to $t = 1$. 4